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The estimation of declining operation returns for industrial property

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The estimation of declining operation returns for industrial property

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INTRODUCTION

Value is a difficult term to define. In whatever context it is used, in whatever discussion it is found, the definition and quantification of value is totally dependent upon the needs, perceptions, and experiences of the parties involved. Since it is impossible to use value in an economic context without defining and quantifying it, much judicial and scholarly effort has been expended over the years to establish acceptable definitions of, and estimation procedures for, value. The objective of this dissertation is to propose a procedure that refines some of the existing procedures for estimating economic value of industrial properties. Application of this procedure will improve the quantification of value for these and other similar properties.

Definitions of Value and Valuation

Before presenting and explaining the proposed procedure, it would be helpful to concisely, simply, and completely define "value" in an economic context. The problem that immediately arises, however, is that no concise, simple, or complete definition exists. Rather, "value" assumes a multitude of meanings depending on how and where the term is used. Bonbright recognized this problem in 1937 when he opened his authoritative treatise with the following passage from Through the Looking Glass, by Lewis Carroll:

> "When I use a word," Humpty-Dumpty said, in a rather scornful tone, "it means just what I choose it to mean--neither more or

> less."
> "The question is," said Alice, "whether you can make words mean so many different things." (1, p. 3)

After using this passage to set the stage, Bonbright then proceeded to discuss, at great length, the many meanings that value may have.

In a text published some 16 years later, Marston, Winfrey, and Hempstead also recognized the many faceted meanings of the term value when they stated that:

> ...literature is replete with such terms as appraised value, assessed value, book value, cost value, earning value, exchange value, fair value, forced sale value, imputed value, intrinsic value, investment value, just value, justified market value, market value, normal market value, normal value, nuisance value, objective value, physical value, rate-making value, real value, reasonable value, replacement value, sale value, scrap value, salvage value, sentimental value, sound value, stock and bond value, subjective value, true value, and value in use. (2, p. 3)

At a much earlier date, Justice Holmes, when referring to the term value in a court decision, said that:

> A word is not a crystal, transparent and unchanged; it is the skin of a living thought, and may vary greatly in color and content according to the circumstances and the time in which it is used. (3)

Finally, the multi-faceted nature of the term value was recognized by Justice Brandeis when he stated that, "value is a word of many meanings" (1, p. 37). Thus, one can easily arrive at the conclusion that a totally adequate definition of value is not a simple thing to present.

Considering the substantial evidence that a singular or all encompassing definition of value is difficult to formulate, the temptation is strong to avoid the task, and refer to value using vague, mysterious, and undefined terms. Proposal of a value estimating procedure, however, necessitates at least a workable definition be presented, even if a completely accurate and satisfactory one cannot be devised. Bonbright summarized his discussion of the many meaning of value with the statement that value was:

> ...either (a) the market value of the property, defined as the price for which the property could actually be sold, or (b) the value of the property to the owner himself. (1, p. 128)

Marston, Winfrey, and Hempstead, after much less discussion of all the possible meanings of value, stated that:

> ...the value of property will be used in the concept of the desirability of ownership or value to the owner... (2, p. 4).

Both of the definitions presented above have two points in common. First, the value of an item is often defined in relation to what some other party or parties, referred to in general terms as the "market", will exchange for that item. Babcock supported this point when he defined value simply as, "...the ratio in which goods exchange" (4, p. 26). Though the standard of exchange may be any number of items, money is the most common.

The second point common to these definitions is that value is often defined in terms of the current owner's conception of the worth of the item to him. Presumably, the owner would relinquish ownership if the market value should rise to or above a point of being equal. Only in unusual circumstances would value to the owner being less than value to the market not result in a sale or exchange of property. One notable exception to this statement would be if the owner was ignorant of current market conditions.

In summary, economic value of a property is best defined as either a monetary measure of worth as determined by an appropriate marketplace, or a monetary measure of the desirability of ownership to the current or potential owner. Where necessary to more clearly define a particular concept of value, a modifying term such as "assessed", "market", "original", or "intrinsic" can be used.

In addition to defining value, the closely related term "valuation" should also be defined. Bonbright defined this term as:

> ...the procedure and technique of estimating the value of specific property at a stated time and place. (1, p. 10)

Marston et al. defined the same term as:

> ...the art of estimating the fair monetary measure of the desirability of ownership of specific properties for specific purposes. (2, p. 1)

In both of these definitions, valuation was described as a procedure for estimating value. Also contained in these two definitions, however, was an important valuation concept. This concept was that the value of a property can be accurately estimated only if the time and purpose of the valuation are known. Based on the preceding definitions, and the desire to define the term as simply as possible, valuation may be defined as the art of estimating value, based on a known time and purpose, in an appropriate and professional manner. Using this definition, the objective of this dissertation may be restated as being the proposal of one or more procedures to refine and improve the valuation of industrial properties.

Evidences of Value

Since value is difficult to define, it is also virtually impossible to precisely quantify it in most cases. Therefore, the valuation engineer or appraiser normally resorts to a number of "evidences" of value to support the ultimate and judgemental determination of value. Bonbright defined four evidences of value when he stated that valuation procedures:

> ...base the valuation (a) directly on actual sales of the same property or of similar property, (b) on the actual cost of the property, (c) on estimated replacement cost with allowances for depreciation, and (d) on a capitalization of income derived or derivable from the property. (1, p. 128)

Marston et al. presented three evidences of value in the following quotation from their text:

> Market price, cost of replacing the service rendered by the property, and present value of the future returns from the property are usually relatively good measures of the value of property to the owner. (2, p. 4)

Explanation of these three evidences reveals that they embody the same principles as the four evidences defined by Bonbright.

Babcock stated that there were seven practical valuation methods. These were:

> Method I, Income Method (commercial rentals); Method II, Income Method (business profits attributable to real estate); Method III, Income Method (business profits allocable to real estate and chattels); Method IV, Income Method (business profits allocable to real estate and business); Method V, Replacement Cost Method (business profits); Method VI, Market Comparison Method (amenity returns); and Method VII, Replacement Cost Method (amenity returns) (5, p. 79)

In the discussion of these seven methods, Babcock acknowledged that they were variations of three general methods of valuation: the "income method", the "replacement cost method", and the "market comparison method" (5).

Further support of the definition of three evidences was provided by the American Society of Appraisers with the following statement:

> In the valuation of Real Estate, there are three acceptable approaches: The Market Data, Income, and Cost Approaches. (6, p. 12)

Based on the authoritative sources cited above, three evidences of value will be defined for later use in this dissertation. These are:

1) Market evidence

2) Income evidence

3) Cost evidence

Each of these requires some discussion and explanation prior to its later application, however.

The Market Evidence

The first of the evidences listed above is the market evidence. Recalling that half of the value definitions presented earlier referred to the property's worth on the open marketplace, one might infer that the market evidence provides the strongest indication of the three evidences. Bonbright supported this inference when he stated, as an opening to his market evidence discussion, that:

> The method of valuation which will now be discussed is given first place, sometimes to the exclusion of all other evidence, in the legal valuation of marketable forms of property. (1, p. 134)

If the market evidence is such an overwhelmingly strong evidence of value, why then does the appraiser even bother to compute the other two evidences? The answer lies in the fact that many kinds of property do not enjoy a readily marketable status. Again quoting Bonbright:

> But only with respect to highly marketable property, and not always even there, is a court or appraiser justified in accepting uncritically the record of current sales as the measure of market

value. (1, p. 136)

The market evidence, therefore, is considered to be a strong evidence of value if the property in question has been frequently traded on the open market. If the property, or similar units, are rarely or never sold for other than salvage value, then the strength of the market evidence diminishes considerably.

The Income Evidence

The second evidence of value is the income evidence. The basis for this approach is that the value of a property is evidenced by the discounted present worth of the anticipated future income stream and future salvage realized. One of the earliest recorded applications of the income evidence to determining value is found in early Jewish law. As stated in Leviticus 25:15,16 of the Bible, the value of a unit of real estate was set "...according to the number of annual crops. The more years there are to run, the higher the price, the fewer the years, the lower, because he is selling you a series of crops" (7, p. 138). More recently, Bonbright described the income evidence as being the result of a two step procedure. "The first step," said Bonbright, "is that of estimating the separate services that may be anticipated, the future dates of their realization, and the value of each service if and when realized. The second step consists of the application to the separate anticipated services of appropriate rates of discount..." (1, p. 219).

The weight to be given the income evidence is dependent not only on the nature of the property, but also on the probability that the estimated future income stream will in fact occur. If the property derives its value predominantly from its ability to produce income, then significant weight should be given to this evidence. Similarly, and as stated by Marston et al., "...when future earnings have a high probability of being realized, earning value is deserving of the major weight" (2, p. 350).

The Cost Evidence

The third evidence of value is the cost evidence. The basis of this approach is that value is evidenced by the adjusted cost of the property. At least four kinds of cost have been identified and defined as the cost to be adjusted. These are the original cost, trended original cost, reproduction cost, and replacement cost. Due to the effects that the passage of time has on the monetary standard used to measure value, most valuations use the trended original cost, reproduction cost, or replacement cost to determine the cost evidence. The American Society of Appraisers further simplified the choice of starting points by stating that:

> ...the terms Reproduction Cost and Replacement Cost are synonymous. (6, p. 13)

Further, the same equipment valuation manual stated that:

> All run-of-the-mill market value appraisals are made under the cost of replacement theory. It is the most simple of approaches. (6, p. 56)

When defined with more rigor, however, there is a distinct difference between reproduction cost and replacement cost. According to Marston et al., replacement cost is "...the estimated cost of replacing the service of the existing property by another property, of any type to achieve the most economical and preferred service, but at prices as of the date specified." Reproduction cost, on the other hand, is "...the estimated cost of reproducing substantially the identical property at a price level as of the date specified" (2, p. 9). For purposes of this dissertation, the distinct differences between replacement cost and reproduction cost were recognized and maintained. Therefore, the trended original cost, replacement cost, or reproduction cost were used as the unadjusted cost new starting point for computing the cost evidence. The choice of which cost to use depends on the availability, reliability, and suitability of the cost information available.

An adjustment to the cost new is made to reflect the fact that the property in question has exhausted part of its total usefulness. This loss in usefulness, and therefore value, is broadly termed depreciation. Literature is replete with definitions and discussions about depreciation and in particular, about its relationship to cost accounting procedures. From a valuation perspective, however, depreciation may be defined simply as "...the loss in value of an item of property resulting from a decrease in its ability or capacity to perform present and future service" (8, p. 10). Once the amount of depreciation has been properly estimated, it is subtracted from the cost new to obtain the adjusted cost. This adjusted cost is the cost evidence figure.

As soon as the three evidences of value (Income, Market, and Cost) have been found, a determination of the property's value can be made. Since it is highly unlikely that the three evidences will agree with each other, some means must be found to weight the evidences. Unfortunately, no mathematical formula exists that adequately afixes those weights. Therefore, valuations must always resort to expert judgement to transform the evidences of value into the final product. "Value," said Marston et al., "is a quality always determined by judgement, not by formula, and so specific weight and factors cannot be given" (2, p. 346). The valuation procedure to be proposed by this dissertation is intended to provide a means of improving this procedure rather than to replace the final judgement of the valuation engineer.

Evidences of Value Applied to Industrial Property

The three most commonly accepted evidences of value were presented and explained in the previous section. Though many valuation situations use all three evidences to arrive at a final estimate of worth, the valuation of industrial equipment normally does not. In most situations, the use of the market evidence is precluded by a lack of sufficient arm's-length sales. The few transactions that have taken place often reflect only a scrap value received for the property. Similarly, the income evidence is often useless because each equipment unit may be only a part of an income producing entity. As such, it is very difficult to estimate what portion of the total income is due solely to the machine being appraised. Unless a future income stream can be estimated, the income evidence cannot be used. Since the market and income evidences of value are insufficient for most industrial equipment valuations, the valuation engineer is left with only one option: the cost evidence. This conclusion was also reached by the Iowa Department of Revenue:

> After considerable study, it was determined that the cost approach should be used to determine the fair market value of industrial machinery and equipment. (9, Preface)

Further support for this conclusion is contained in the American Society of Appraiser's manual on industrial equipment valuation:

> In conclusion, it can be safely stated that Reproduction Cost New, with adjustments based on factual data, can serve as a most effective tool to establish Fair Market Value of machinery and equipment. (6, p. 16)

Therefore, since the cost evidence of value is the most reliable evidence available for industrial equipment, it will become the cornerstone of the value determination procedure to be proposed.

REVIEW OF PREVIOUS WORK

The valuation procedure to be proposed by this dissertation is not a pioneering effort in the field of engineering valuation. Rather, it is a refinement to existing valuation procedures that have developed over a long span of years. To put this refinement in proper context, a brief summary of significant or previous developments in closely related areas of valuation will be presented.

Sinking Fund Method of Estimating Depreciation

When computing the cost evidence to be used in an industrial equipment valuation, estimating the amount of depreciation is often the largest potential source of error in the calculation. Though the cost new for a property item or group can be found with reasonable accuracy in most cases, the allocation of depreciation over the property's lifespan is not easily determined. Winfrey commented on the difficulties of determining depreciation when he said that:

> The handling of depreciation has caused much controversy because of the difficulty, first, of determining the total depreciation in advance of retirement, and second, of determining the time distribution of depreciation in order to measure in dollars this element in the total cost of operation. (8, p. 18)

Having stated that the depreciation estimate is difficult to determine accurately, Winfrey then stated that:

> There are only three methods within reasonable bounds of plausible theory by which depreciation should be estimated.... These three methods are: the straight-line assumption, the sinking-fund assumption, and the present- worth principle. (8, p. 19)

Since the straight-line assumption is easily understood and readily applied, and the present-worth principle will be covered in some detail in a later section, only the sinking-fund assumption will be discussed at this point.

Determining depreciation using the sinking-fund assumption is based on the premise that:

> ...the accrued depreciation of a property unit to any date is equal to the accumulation in a ficticious equal-annual-year-end-payment sinking fund, in which the total accumulation at retirement of the unit equals the depreciable value new of the unit. This assumption also takes into consideration the probable life and salvage value of the unit. (8, p. 20)

Converting this definitional statement into a mathematical equation results in the following expression for the accumulated depreciation at any age:

$$f_x = V_{nd} \frac{(1+i)^x - 1}{(1+i)^n - 1} \qquad (1)$$

where, f_x = accumulated depreciation

V_{nd} = depreciable value new

i = annual interest rate

x = age of the unit in years

n = probable life of the unit in years.

The expression for the present value, which is equal to the cost evidence, is as follows:

$$V_p = V_{nd} \frac{(1+i)^n - (1+i)^x}{(1+i)^n - 1} + V_s \qquad (2)$$

where V_{nd} = present value, V_s = salvage value, and the other symbols are as previously defined. The detailed derivation of these expressions was presented by Winfrey (8), Marston and Agg (10), and Marston et al. (2).

The significance of the sinking fund method for determining depreciation is that it was one of the earliest methods allowing incorporation of an annual interest rate into the depreciation calculations. Though this characteristic is an improvement, Marston and Agg refer to the sinking fund method as "...purely ficticious; merely a mathematical concept...", applying only in "...an equal-annual-year-end-payment sinking fund...", and using interest rates "...fixed by arbitrary custom..." (10, p. 100). Though exception to their conclusion is expressed by Bonbright (1, p. 192-193), Marston and Agg do conclude that the primary failing with the sinking fund method is that the discount rate is too low. Rather than basing the discount rate on a reasonable rate of return, Marston and Agg point out that the rate was arbitrarily set at three to five percent.

Present Worth Method of Estimating Depreciation

In addition to the straight-line assumption and the sinking-fund assumption, depreciation can be estimated using the present-worth principle (8, p. 19). Placed in a historical context, this principle is simply another in a series of depreciation methods proposed since depreciation theory began developing about the turn of the century. When compared to other depreciation estimating methods, however, the present worth principle is considered to be the only procedure based on sound theory rather than arbitrary assumptions (10, p. 105; 2, p. 198; 8, p. 21; 11). Its development was significant not only to valuation theory as a whole, but to this dissertation in particular.

The Present-worth Actual Depreciation Principle

As proposed by Marston, the present-worth principle was stated as follows:

> The present-worth actual depreciation principle is that the depreciated value of an industrial-property unit, at any date during its service life, is the present worth at that date of the probable future operation returns yet to be earned by its probable future services. (10, p. 105)

Though it is not clearly stated in the quotation above, the probable future services included the net salvage value realized at the point of retirement as well as the future operation returns.

The operation returns of a property unit were defined as including both the periodic depreciation and the net return on the depreciated value (8, p. 25). Referred to as the after-tax cash flow, another source defined this term as including the interest on debt capital to be paid, the depreciation expense, and the net income (12, p. 93). In the latter case, a valuation rather than cost accounting definition must be applied to the depreciation portion of the term.

The present-worth principle was restated in a slightly different form by Marston et al. at a later date. According to this source, the present-worth principle was:

> ...the value of a property, at any date during its service life, is the present worth at that date of the probable future operation returns yet to be earned through its probable future services. (2, p. 198)

The operation returns referred to in this quote are end of period returns.

In the valuation refinement to be proposed, the present worth principle was the best means of determining the adjusted reproduction or replacement cost of the property. Using the present worth principle, the adjusted value can be found directly. This has been found to be superior to the more traditional procedure of finding reproduction cost new, determining a depreciation amount, then calculating an adjusted reproduction cost as the difference between those two amounts (2, p. 181,121). The traditional procedure is more appropriate in a cost accounting situation rather than a valuation situation.

Mathematical Expression of the Present-worth Principle

Marston's derivations of the mathematical equations associated with the present-worth principle were based on discounting the anticipated future annual operation returns, and the estimated salvage value at retirement, for the property units in question. The discount rate to be used was specified as the fair rate of return for the property. The resulting equation for the present value was (10, p. 109):

$$V_p = V_{nd} \frac{(1+r)^n-(1+r)^x}{(1+r)^n-1} + V_s \qquad (3)$$

where, V_p = the unit's present value at age x

V_{nd} = the unit's depreciable value new

V_s = the unit's salvage value at retirement

r = the fair rate of net return on the entire property

x = the unit's service age in years

n = the unit's probable life in years

Winfrey's derivation in a later source (8) was more detailed, but resulted in the same equation.

Marston also derived an equation for the total accrued depreciation.

This equation was as follows (10, p. 110):

$$D_p = (V_{nd})(1 - \frac{\text{condition percent}}{100})(\text{PFORR}) \qquad (4)$$

where, D_p = the unit's present total accrued depreciation at age x

V_{nd} = the unit's depreciable value new

PFORR = the unit's probable future operation-return ratio for the probable future life n-x

$$\text{condition percent} = \frac{(1+r)^n-(1+r)^x}{(1+r)^n-1}\ (100)$$

Both the PFORR and the condition percent terms will require more definition and explanation. The condition percent will be discussed next; the PFORR term in a later section.

As defined by Marston and Agg, the condition percent term, expressed as a percentage, is "...100 times the ratio of its present depreciable value divided by its depreciable value when new" (10, p. 37). Marston and Agg continued their definition by noting that four differing applications of the term exist. These are the condition percents for: 1) a single property unit, 2) an average survivor unit of an age-group, 3) all survivors of an age-group, and 4) all units in service from an age-group where continued renewals maintain a constant population (10, p. 37). Though the basic definition of the condition percent does not change for these applications, the method of computing the condition percent value does.

Winfrey defined the condition percent in terms similar to Marston and Agg in a later publication (8, p. 26). Tables giving the condition percent values for properties at different ages for differing probable lives and discount rates, and based on the assumption of uniform annual operation returns, were also published at a later date (13). In another later publication co-authored by Marston and Winfrey, the condition percent term was called the "expectancy-life factor" (2, p. 200).

When derived for a unit of property, the mathematical expression for the condition percent factor is:

$$C = \frac{(1+r)^n-(1+r)^x}{(1+r)^n-1} \tag{5}$$

where, C = condition percent factor (as a decimal)

r = annual net rate of return

n = probable life of unit in years

x = age of unit in years.

In common usage, the condition percent factor (C) is often multiplied by 100, thereby expressing it as a percentage, and referred to simply as the condition percent (C_p). Further discussion of condition percent factor characteristics were given by Marston and Agg (10), Winfrey (8), and Marston et al. (2).

Non-uniform Annual Operation Returns

Though the present-worth principle, as stated earlier, does not depend on the existence of a uniform annual operation return stream, the derivations of Equations 3, 4, and 5 are based on this simplifying assumption (8, p. 31; and 2, p. 199). Convincing evidence exists, however, that operation returns decrease with age rather than remaining constant.

Recognition of Decreasing Operation Returns The existence of decreasing operation returns was acknowledged as long ago as 1916 by Campbell when he stated that "...physical deterioration is generally a source of increasing contingent loss" (11, p. 13). Later, Terborgh published a book containing analyzed data from several sources having particular relevance to this dissertation. Though the primary thrust of Terborgh's book concerned equipment replacement policy, he included two sets of graphical charts showing a decrease in the quantity of measured service and an increase in repair costs as equipment ages. These two sets of charts are reproduced in Figures 1 and 2 respectively. The decreasing quantity of service and increasing repair costs translated directly into a decreasing stream of operation returns, or after-tax-cash-flows, over the lifespan of an equipment unit. Both the sinking fund method, by definition, and the present worth method, by practice, utilized a constant level of operation returns to compute the amount of accrued depreciation. Therefore, Terborgh's results were in direct conflict with those methods. Because this dissertation is based in large part on the existence of a decreasing stream of operation returns, Terborgh's previous work is supportive of the basic assumptions used to derive the proposed procedures.

Further support of the existence of a decreasing operation returns stream was given by Marston et al.. As presented by this source, the decrease was "...caused by lowered efficiency of the property, lowered output capacity, increased running costs, intermittent (stand-by) service, and operation at less than normal capacity" (2, p. 182). The existence of decreasing, as opposed to constant, operation returns has therefore been recognized by a number of authoritative sources for many years.

<u>Methods of Handling Decreasing Operation Returns</u> The existence of decreasing operation returns has resulted in the need for some means of adjusting the computed present value. Past methods for handling this nonuniformity have varied. Marston and Agg introduced a "probable future operations return ratio" (PFORR) into their equation for present value. The resultant expression then became:

$$V_p = (V_{nd})\left(\frac{\text{condition percent}}{100}\right)(\text{PFORR}) + V_s \tag{6}$$

where all terms are as previously defined (10, p. 109). The expression for accrued depreciation remains the same as Equation 4 since the PRORR was already incorporated in that equation.

In a later source, Marston et al. defined a similar term, but called it a "service factor". In discussing the service factor, this source stated:

> The service factor may be less than unity or greater then unity; its main function is to compensate, when necessary, for failure of the expectancy-life factor to produce the desired adjustment of the base new to current conditions. (2, p. 236)

The service factor was then inserted into Equation 3 in the same manner and with the same resulting expression as the PFORR term in Equations 4 and 6.

Determination of a value for the service factor or PFORR was "...one of judgement to be introduced as the appraiser may see need for its use" (2, p. 235). Due to the very subjective method of selecting this value, most appraisers and valuation engineers have simply ignored the term (or equivalently, set the value to unity) in the equation. The result has been to carry the initial assumption of a uniform annual operation return stream through to the final cost evidence determination.

An alternate method for handling the nonuniform operation returns was proposed by Elfar (14). This method incorporated a progression rate term, called a "T-factor", into the derivation of the condition percent factor. The T-factor was related to the operation return stream as shown in the following expression (14, p. 48):

$$R_x = R_1\left[\frac{T^N - T^{x-1}}{T^N - 1}\right] \qquad (7)$$

where, N = probable life of property unit or frequency group in half-year intervals

x = age of unit or property group in half-year intervals

R_x = operation return for age interval (x-1) to x and treated as an end-of-interval quantity

T = progression rate of operation returns.

The purpose of the T-factor was to incorporate the effects of a variety of operation return streams, including the uniform case, into the present worth determination of value. The result of the derivation was a "modified condition percent factor" that was used in a manner similar to the original condition percent factor. The modified condition percent factor, however, combined the original condition percent factor with the service factor or PFORR term.

Due to its importance to the central topic of this dissertation, a summary and brief discussion of the equations derived by Elfar are presented. A detailed derivation of the valuation model can be found in either the Elfar thesis (14) or the Cowles-Elfar paper (15).

Beginning with the present-worth principle as stated by Marston, and incorporating Equation 7 at the appropriate point, Elfar derived the following expression for the value at any age (14, p. 51):

$$V_X = V_N\left[C'_x(1-S) + S\left[C'_x(1-(p/f)^i_N) + (p/f)^i_{N-x}\right]\right] \quad (8)$$

where, V_x = value at age x of a unit or of survivors of a property group

V_N = value new of a unit or of survivors of a property group

C'_x = modified condition percent factor at age x

S = salvage ratio = (V_S / V_N)

V_S = estimated net salvage value

$(p/f)^i_N$ = present worth of a future sum

i = effective semi-annual discount rate

N = probable life of property unit or frequency group in half-year intervals

As indicated by the units for several of the variables identified above, Elfar's derivation was in terms of half-year intervals. In the interest of simplicity and uniformity, this convention will be carried forth in this dissertation. The effect of using whole-year intervals, since they are more commonly used in valuation practice, will be explored later.

The "modified condition percent factor" contained in Equation 8 was analogous to the condition percent factor derived by Marston, Winfrey, and others (10, 8, 2). The significant difference was that the condition percent factor was a special case of the more general modified condition

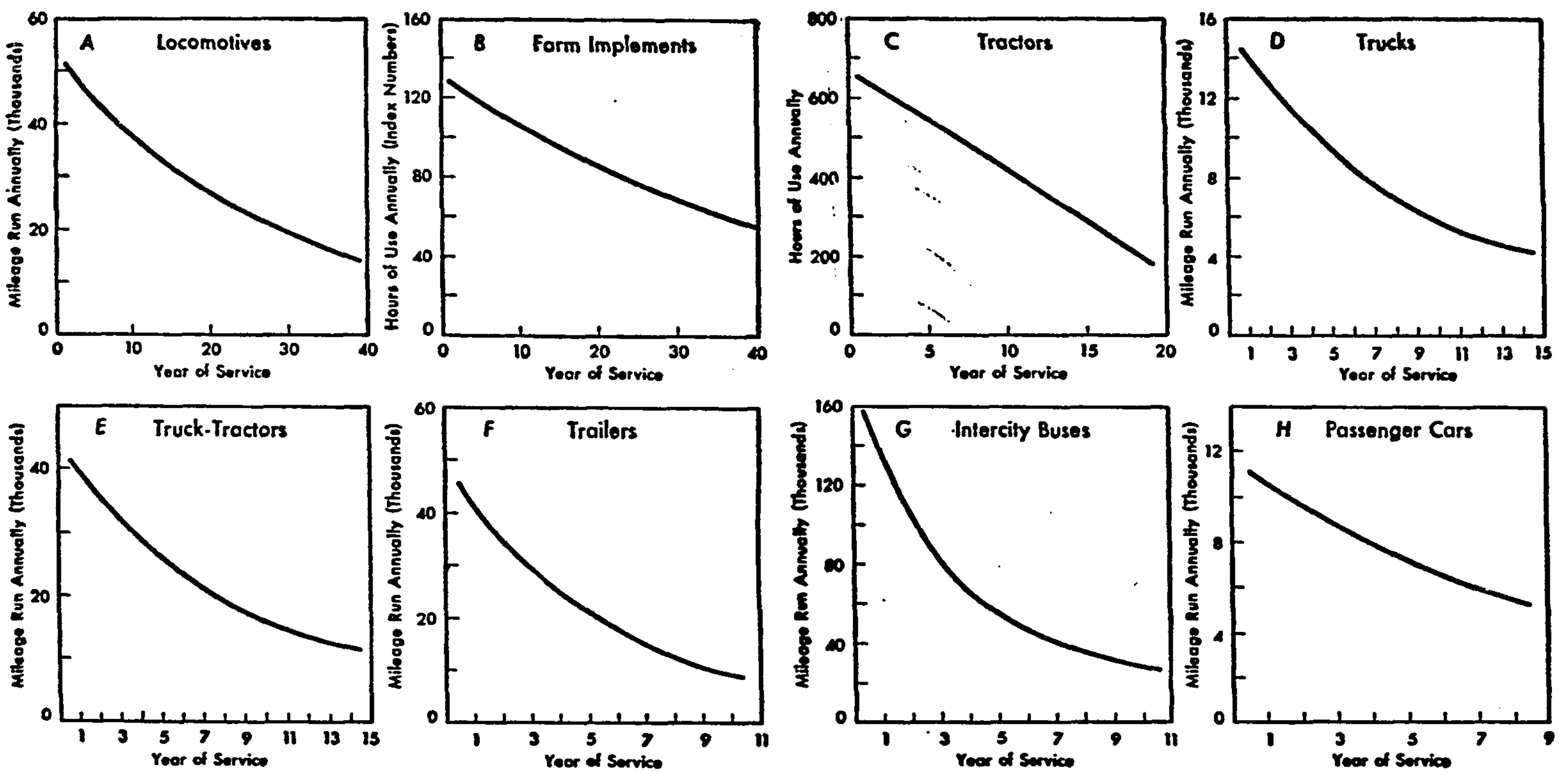

Fig. 1. Relation between age and intensity of use of eight classes of equipment (16)

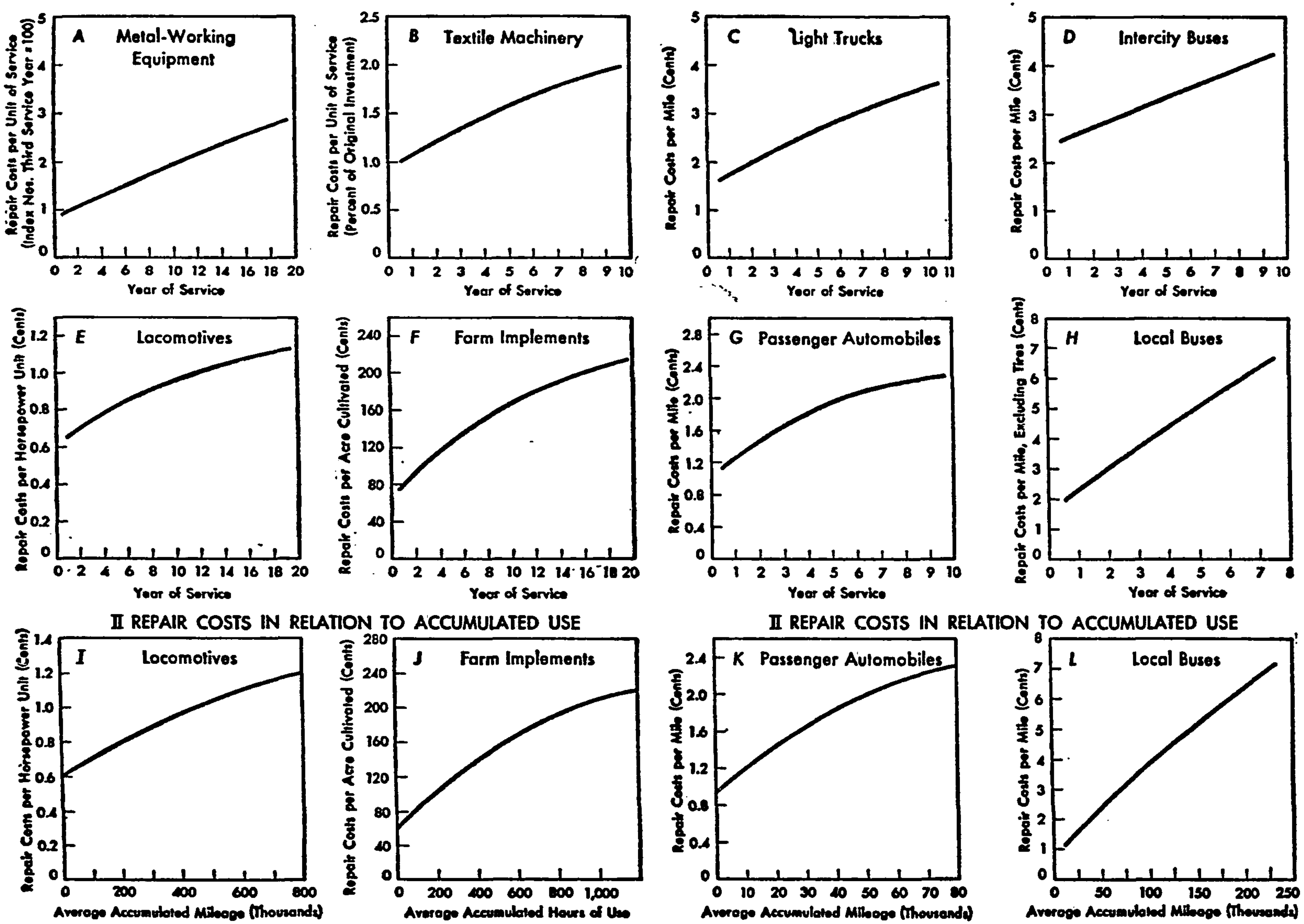

Fig. 2. Relation between age and repair cost per unit of service for twelve classes of equipment (10)

percent factor. As derived by Elfar, the general, closed form, mathematical expression for the modified condition percent was:

$$C'_x = \frac{q^{N-x-1}(T + iT^{x-N}) - q^{-1}(T + i) - q^{N-x} + 1}{q^{N-x-1}(T + iT^{-N}) - q^{-x-1}(T + i) - q^{N-x} + q^{-x}} \tag{9}$$

where, $q = (1+i)$ and all other terms are as previously defined (14, p. 55).

By definition, the progression rate, T, was in the range of $0<T<\infty$. Likewise, the semi-annual rate of return, i, was some value in the range of $0<i<\infty$. Within these two ranges, however, the two terms may assume values that resulted in unique forms of the general expressions given in Equations 8 and 9. These special cases, as defined by Elfar (14, p. 55), occurred when:

$T = 1$ and $i > 0$

$T = \infty$ and $i > 0$

$T = 1$ and $i = 0$

$T = \infty$ and $i = 0$

$T < \infty$ and $i = 0$

The resulting, special case, expressions are presented and briefly discussed below.

<u>$T = 1$ and $i > 0$</u> If $T = 1$, then the operation returns declined by an equal amount each period. The resulting equations for the modified condition percent factor and the value at age x were:

$$C'_x = \frac{q^{N-x}(1 + ix - iN) - 1}{q^{N-x}(1 - iN) - q^{-x}} \tag{10}$$

$$V_x = V_N\left[C'_x(1-S) + S\left[C'_x(1-(p/f)^i_N) + (p/f)^i_{N-x}\right]\right] \tag{8}$$

where, all terms are as previously defined (14, p. 56, 58).

$T = \infty$ and $i > 0$ A T-factor equal to infinity represented a situation where operation returns did not decline from year to year, but remained uniform instead. This corresponded to the basis of the original condition percent factor as defined by Marston and Agg (10). The general equations for the modified condition percent factor and the value at any age reduced to the following expressions for the $T = \infty$ case (14, p. 59, 60):

$$C'_x = \frac{(1+i)^N-(1+i)^x}{(1+i)^N-1} \tag{11}$$

$$V_x = (V_N-V_S)C'_x + V_S \tag{12}$$

As before, all terms are as previously defined. Comparison of Equations 11 and 12 to similar results in the Marston and Agg (10), Winfrey (8), and Marston et al. (2) references verified the uniformity of results. These previous derivations and definitions, therefore, were but a special case of the model presented by Elfar (14).

$T = 1$ and $i = 0$ A value of $T = 1$ represented, as explained in a previous section, the linear decline of operation returns. By specifying a value of $i = 0$, however, this case also represented a zero rate-of-return on investment situation. The resulting special case equations were (14, p. 63, 64):

$$C'_x = \frac{(N-x)(N-x+1)}{N(N+1)} \tag{13}$$

$$V_x = (V_N-V_S)C'_x + V_S \tag{12}$$

where the terms are as previously defined.

T = ∞ and i = 0 This special case represented a uniform operation return stream under the influence of a zero rate-of-return on the investment. The resulting equations for this case were as follows (14, p. 65):

$$C'_x = \frac{N-x}{N} \tag{14}$$

$$V_x = (V_N - V_S)C'_x + V_S \tag{12}$$

These equations were identical to results obtained for similar conditions by Winfrey (8).

T < ∞ and i = 0 The final special case represented a zero rate-of-return condition with no specified value for the T-factor. If T = 1 or ∞, then this special case further reduced to equations presented earlier. The specialized equations for this case were (14, pp. 61, 62):

$$C'_x = \frac{T^{N+1}\left[(T-1)(N-x)-1\right] + T^{x+1}}{T^{N+1}\left[(T-1)N-1\right] + T} \tag{15}$$

$$V_x = (V_N - V_S)C'_x + V_S \tag{12}$$

By selecting the correct general or special case equation, the modified condition percent factor and corresponding value at any age could be calculated based on the present-worth principle.

Depreciation Due to Obsolescence

In addition to depreciation resulting from the causes previously listed, depreciation also occurs due to economic and functional obsolescence. This contribution to depreciation was recognized by Marston and Agg when they listed obsolescence as one of the seven causes of property retirements (10, p. 83). (Marston and Agg had previously stated that the main cause of depreciation was the inevitable approach of the property's retirement date.) The existence of obsolescence as a contributor to depreciation was also recognized by Terborgh (16). Additionally, Terborgh described the nature of obsolescence when he stated:

> ...obsolescence...is usually more or less 'lumpy' or irregular, reflecting sudden changes in the product or improvements in the currently available alternatives. There is no reason, however, to suppose that such sudden changes are more likely to occur at one stage than another in the lives of the machines adversely affected. Overall, obsolescence probably represents a fairly steady and continuous pressure. From the standpoint of probability or prediction, therefore, unless there is special reason to deviate from the pattern in the particular case, the most reasonable assumption is that obsolescence is a risk, indeterminately over time, hence that its incidence is random. (16, p. 67)

The characterization of obsolescence as a continuous and constant source of depreciation greatly simplifies its incorporation into valuation computations.

The actual quantification of obsolescence is sometimes a difficult achievement even with Terborgh's simplification, however. One rather simple, alternative quantification procedure was presented by Saliers in his definition of obsolescence:

> If the first cost of the old and the new plants, respectively, be added to the capitalized cost of operating them, the difference, assuming that both perform the same service, represents the depreciation of the old plant from obsolescence. (17, p. 29)

Another procedure was proposed by K. J. Affanasiev (18, pp. 345-9). This procedure was based on comparing the operating expenses of an older, existing property unit against a newer, improved unit. The result of Affanasiev's derivation was the following expression:

$$P_o = V_N - V_N' + (e-e')(p/a)_n^i \qquad (16)$$

where, P_o = obsolescence amount

V_N = original cost of the existing property

V_N' = original cost of the improved property

e = annual operating expense of existing property

e' = annual operating expense of improved property

Though the simplicity of the expression above may be questionable, the reasoning behind the derivation of Equation 16 was not only sound then, but remains particularly applicable to this dissertation. In the accompanying explanation, Affanasiev stated:

> ...the extent to which the cost of production or of service can be lowered by replacing old units with improved units, or the extent to which the quality and reliability of service can be improved at substantially the same cost as or less cost than the cost of operating the old units, measures depreciation due to obsolescence in the existing units. (18, p. 346)

Quantifying obsolescence in this manner closely conforms with the present worth principle presented earlier. The incorporation of obsolescence, whether it is measured directly or estimated using a technique similar to Affanasiev's, has been recognized as essential to the accurate determination of depreciation.

OBJECTIVES

The Elfar valuation model presented in the last division is complete in its derivation. In addition, Elfar discussed the effects of varying the parameters contained in the model, and verified that the Iowa type curves did apply to industrial properties (14). Though values for most of the parameters can be estimated using previously proven techniques, no procedure for estimating a suitable value for the progression rate T has been proposed. Before the Elfar model could be applied to actual valuation situations, a procedure had to be devised to estimate the value of T exhibited by the property in question. The need for this estimation procedure was the principal reason for undertaking this study. As this need evolved into a specific project, four intermediate objectives were defined as being necessary to the solution of the overall problem. These four objectives were:

1) Identification of a small number of select property accounts with substantial market evidence data,
2) Determination of an appropriate T-factor for these select property accounts,
3) Derivation of a procedure to estimate T-factor values for more general cases, and
4) Development of a set of standard curves for use in valuation situations.

The first and second objectives listed above were necessary for several reasons. One such reason was that a T-factor value could only be determined, unless the procedure to be proposed later was used, for properties having a substantial market evidence curve. For reasons presented earlier in this dissertation, industrial properties with this characteristic are virtually nonexistent. Indeed, if they were widespread, there would be no need for the valuation model proposed by Elfar. Therefore, select, but similar, property groups with market evidence curves were identified, and data was collected for the analysis and development steps that followed. A second reason for determining T-factor values for a select group of properties was so that a control group of properties would be available to test the proposed estimating procedure against. Without a comparison of results obtained from the proposed procedure to results obtained from other, previously proven methods, no evaluation of accuracy could be made. A final reason for finding T-factor values for a select group of properties was to obtain some preliminary verification that Elfar's valuation model was valid. If no T-factor value could be found that closely duplicated an observed market evidence curve, then one might reasonably conclude that the Elfar valuation model was not valid. The study could then have been directed toward the proposal of an alternate or modified model rather than the original purpose.

The third objective of the study was to derive a procedure for estimating T-factor values for more general cases. Application of the Elfar valuation model to general valuation situations was not possible without this objective being accomplished. Therefore, this objective was considered to be the most important of the four objectives listed. It was, in fact, the primary objective of this dissertation.

The fourth objective, development of a set of standard curves, was considered desirable if the proposed procedure was to be simply and quickly applied. The T-factor value estimated using these standard curves could be quickly and easily substituted into the appropriate equation to obtain an improved cost evidence of the property's value at any age.

ESTIMATION PROCEDURE DEVELOPMENT

Development of a general procedure to estimate T-factor values was dependent upon finding a relationship between the rate at which operation returns decrease and the rate at which depreciation accrues. The rate of decrease in operation returns was reflected in the estimated T-factor value whereas the amount of depreciation accrual was reflective of the value at any age. Two approaches were developed. The first of these, referred to as the "Ratio Procedure", attempted to relate a ratio of the operation returns at the end of the first interval (R_1) and at the end of the interval (x-1) to x (R_x) to the accrued depreciation amount at age x. The second approach, referred to as the "Delta Procedure", attempted to relate the differences of the operation returns at the end of intervals 1 and x to the accrued depreciation amount at the end of interval x. The derivation and explanation of these two procedures are presented below.

Prior to the presentation of these two procedures, however, a list of symbols will be presented. The following list of symbols will be used for the remainder of this dissertation:

N = probable life of property unit or frequency group in half-year intervals

x = age of unit or property group in half-year intervals

V_N = value new of unit or of survivors of a property group

V_x = value at age x of a unit or of survivors of a property group

V_S = estimated net salvage value after N intervals

S = salvage ratio = (V_S/V_N)

R_x = operation return for age interval (x-1) to x and treated as an end-of-interval quantity

r = effective annual dicount rate

i = effective semi-annual discount rate = $(1 + r)^{0.5} - 1$

T = progression rate of operation returns

$(p/f)^i_N$ = present worth of a future sum = $(1 + i)^{-N}$

$(p/a)^i_N$ = present worth of a uniform series = $\frac{(1+i)^N-1}{i(1+i)^N}$

$(a/p)^i_N$ = uniform series worth of a present sum or capital recovery factor

= $1/(p/a)^i_N$

Ratio Procedure

The Ratio Procedure proposed was based on the premise that a decreasing ratio of operation returns was related to an increasing depreciation accrual rate. The operation returns ratio was specified as being R_x divided by R_1. The actual procedure derived was based on discussions and information obtained from a major oil production and refining company (19,20). A concept explanation, procedure derivation, and example calculation for the Ratio Procedure are presented in the following subsections.

Explanation of the Concept

The proposed Ratio Procedure was based on the measurement of after-tax net returns for an existing and a new refinery. The returns measured were in units of net dollars per year. A ratio of the existing refinery's net returns to the new refinery's net returns was then formed.

The resulting ratio was then set equal to a function of the progression rate in accordance with Elfar's basic model. Knowing the estimated amount of the net returns and the probable life, the estimated T-factor was then calculated for a specified age.

One clear distinction to be made, however, is that the new refinery returns were based on a replacement property rather than a reproduction property. That is, the new refinery included all of the technological advances that had occurred since the existing refinery was built. By using a replacement property rather than a reproduction one, the most significant component of depreciation--economic obsolescence--was automatically included in the calculations. When measured in the manner described above, the existing refinery returns corresponded to the R_x term and the new refinery returns to the R_1 term as defined in Elfar's model.

Derivation of the Ratio Procedure

Applying the concept explained above, the ratio was written as:

$$\text{RATIO} = R_x/R_1 \tag{17}$$

Recalling, and slightly rearranging, Equation 7, the RATIO expression became:

$$\text{RATIO} = R_x/R_1 = \left[\frac{T^N - T^{x-1}}{T^N - 1}\right] \tag{18}$$

where all variables are as previously defined. Since it was possible to estimate reasonably accurate values for R_x, R_1, and N, the refinery's T-factor value could then be computed by trial and error. The results of the Ratio Procedure, as demonstrated by the example in the next subsection, were both consistent and reasonable. The validity of the procedure, when applied to the appropriate situation having adequate data available, was therefore considered to be excellent.

<u>Example of the Ratio Procedure</u>

For illustrative purposes, three oil refineries were analyzed using the Ratio Procedure to determine the best T-factor values. Sufficient data were available for one of the refineries to allow an analysis at three different ages. Therefore, five refineries were in effect available for analysis. As a means of assuring anonymity, the five installations were designated simply as: 1) Alpha-I, 2) Alpha-2, 3) Alpha-3, 4) Beta, and 5) Gamma. The three "Alpha" refineries were the same installation analyzed at three different ages.

The trial and error application of the Ratio Procedure to estimate the T-factor value is shown in Table 1. Supporting calculations for the R_1 and R_x values used in Table 1 are contained in Appendix A. The best T-factor value estimated for each refinery is summarized in the last column of Table 1. All estimated T-factor values fell within a range of .99 to 1.03, with T = 1.01 being the single best estimate. The modified condition percent factors and the values at any age were calculated using Equations 8 and 9, or appropriate special case forms, for the refinery using the resulting estimated T-factor value. The values obtained using the Ratio Procedure compared favorably to the best valuations performed using more conventional methods.

TABLE 1. Estimation of T-factor for oil refineries using the Ratio Procedure

Refinery name	R_1	R_x	x	N	RATIO	Computed ratio value for T=?						Best T-factor value
						T=.98	T=.99	T=1.00	T=1.01	T=1.02	T=1.03	
Alpha-1	14.7	12.0	10	40	.8163	.7001	.7388	.7750	.8084	.8385		1.01
Alpha-2	14.7	8.9	20	42	.6054	.4427	.4952	.5476	.5989	.6479		1.01
Alpha-3	14.7	6.2	32	44	.4218	.2097	.2510	.2955	.3422	.3903	.4385	1.03
Beta	23.2	11.0	20	42	.4741	.4427	.4952					0.99
Gamma	61.7	17.8	34.6	45.6	.2885	.1814	.2205	.2632	.3086			1.01

Delta Procedure

The Delta Procedure is an alternate estimating procedure to the Ratio Procedure presented in the preceding section. Though the Ratio Procedure was found to estimate T-factor values with at least a reasonable degree of accuracy, it was limited to valuations where an accurate estimate of annual operation returns was available. Since adequate information to apply the Ratio Procedure was not normally available for industrial equipment, a more generally applicable procedure was needed. The Delta Procedure was therefore proposed as an alternative procedure for estimating T-factor values.

Explanation of the Concept

At the instant of installation, each piece of industrial equipment produces an idealized level of gross revenue. This idealized level of gross revenue, denoted as G, is assumed to remain constant throughout the property's life. If the unit in question incurred no increase in nonservice downtime, suffered no decrease in production rates, accumulated no obsolescence, and had no increase in repair or maintenance costs, then the net operating return would remain at some constant level below G. The difference would be equal to the fixed, constant cost of operation. This situation would represent the uniform operation return case defined and described previously.

In reality, the net operation returns are reduced each period by an increasing amount. A point is eventually reached where this diminution cannot be tolerated and the property is then retired. If the annual reduction in operation returns is denoted as P, and the corresponding net operation return as R, then by definition,

$$G = P_x + R_x \qquad (19)$$

for any age x. A schematic diagram showing this relationship is shown in Figure 3.

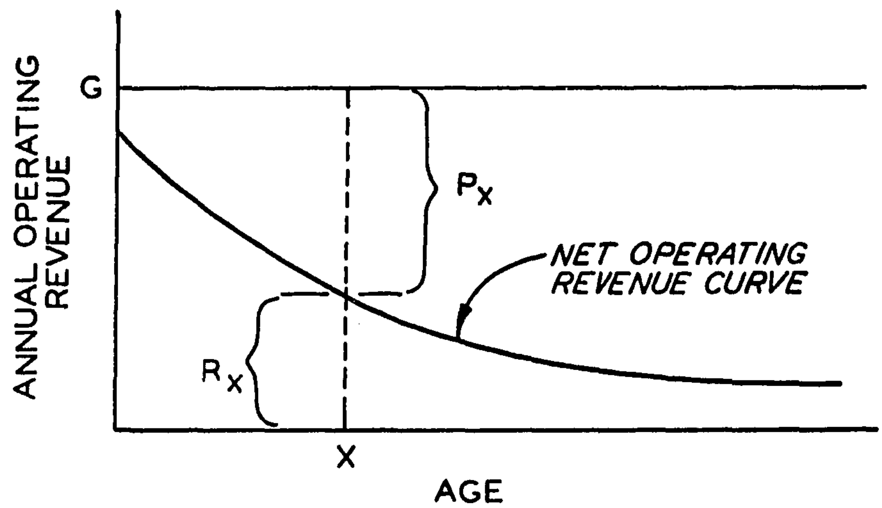

Figure 3. Schematic diagram of Delta Procedure concept

The basic premise of the Delta Procedure is that, though the operation returns cannot be measured, the annual values of P_x can be at least estimated. This yearly reduction is the result of most of the cumulative causes of the decline in value. As has been previously verified from the Marston et al. reference (2), value depreciation results from such causes as increasing repair and maintenance expenditures, decreasing production rates, reduced availability, and accumulating obsolescence. Cost accounting systems for most companies routinely record the amounts of some of these causes. Using this information, the annual reduction in operation returns for successive years can be estimated. This estimate should contain as many of the causes of depreciation as can be reasonably quantified. The rate at which this reduction increases is directly related to the T-factor value exhibited by the property. The nature of this relationship is the subject of the following derivation.

Derivation of the Delta Procedure

The starting point for the Delta Procedure derivation was Equation 19 and Figure 3. Based on these, the value of G at the end of the first interval was expressed as:

$$G = P_1 + R_1 \tag{20}$$

Equations 19 and 20 were then combined to give:

$$G = P_x + R_x = P_1 + R_1 \tag{21}$$

Eliminating the unknown term G from this equation, and rearranging the remaining equation, resulted in the following expression:

$$P_x - P_1 = R_1 - R_x \tag{22}$$

If the quantity $(P_x - P_1)$ is defined as Δ, then:

$$\Delta = R_1 - R_x \tag{23}$$

Substituting Equation 7 into Equation 23 resulted in the following equation:

$$\Delta = R_1 - R_1\left[\frac{T^N - T^{x-1}}{T^N - 1}\right] \tag{24}$$

Drawing from Elfar's derivation, the operation return at the end of the first interval was (14, p. 49):

$$R_1 = \frac{V_N(1-S(p/f)_N^i)(T^N-1)}{\sum_{m=1}^{N}(T^N-T^{m-1})(p/f)_m^i} \tag{25}$$

Minor simplification of Equation 24, and substitution of Equation 25 into Equation 24 resulted in the following equation:

$$\Delta = \left[\frac{V_N(1-S(p/f)_N^i)(T^N-1)}{\sum_{m=1}^{N}(T^N-T^{m-1})(p/f)_m^i}\right]\left[1 - \frac{T^N-T^{x-1}}{T^N-1}\right] \tag{26}$$

Both sides of the equation were then multiplied by T/TV_N, to obtain the following equation:

$$\frac{\Delta}{V_N} = \left[\frac{(1-S(p/f)_N^i)(T^N-1)(T)}{\sum_{m=1}^{N}(T^{N+1}-T^m)(p/f)_m^i}\right]\left[\frac{T^{x-1}-1}{T^N-1}\right] \tag{27}$$

The summation term contained in the denominator resulted in an unwieldy expression that could be greatly simplified with a closed form of the equation. This simplification was initiated by splitting the summation portion into the two terms shown below:

$$\sum_{m=1}^{N}(T^{N+1}-T^m)(p/f)_m^i = T^{N+1}\sum_{m=1}^{N}(p/f)_m^i - \sum_{m=1}^{N}T^m(p/f)_m^i \tag{28}$$

Next, the general expression (21)

$$\sum_{k=1}^{N}aQ^{k-1} = \frac{a(Q^N-1)}{(Q-1)} \tag{29}$$

was modified by multiplying both sides by Q/A to obtain the following equation:

$$\sum_{k=1}^{N}Q^k = \frac{Q(Q^N-1)}{(Q-1)} \tag{30}$$

Noting that $(p/f)^i_N = (1 + i)^{-N} = q^{-N}$, then applying Equation 30 to Equation 28 with $\Omega = T/q$, resulted in the following:

$$T^{N+1}\sum_{m=1}^{N}(p/f)^i_m - \sum_{m=1}^{N}T^m\frac{1}{(1+i)^m} = \left[T^{N+1}\sum_{m=1}^{N}(p/f)^i_m\right] - \left[\frac{Tq^{-1}(T^Nq^{-N}-1)}{Tq^{-1}-1}\right] \quad (31)$$

Further, it was noted that

$$\sum_{m=1}^{N}(p/f)^i_m = (p/a)^i_N = \frac{(1+i)^N-1}{i(1+i)^N} = \frac{q^N-1}{iq^N} \quad (32)$$

Substituting Equation 32 into Equation 31 gave the final simplification of the denominator summation term as follows:

$$\left[T^{N+1}\sum_{m=1}^{N}(p/f)^i_m\right] - \left[\frac{Tq^{-1}(T^Nq^{-N}-1)}{Tq^{-1}-1}\right] = (T^{N+1})\left[\frac{q^N-1}{iq^N}\right] - \left[\frac{Tq^{-1}(T^Nq^{-N}-1)}{Tq^{-1}-1}\right] \quad (33)$$

Substituting this final expression into Equation 27 and simplifying the result gave the following:

$$\frac{\Delta}{V_N} = \frac{(q^N-S)(T^{x-1}-1)(T-q)(i)}{T^N(Tq^N-T-q^{N+1}+1) + (iq^N)} \quad (34)$$

Equation 34 was the closed form, general expression used to estimate the value of T for a given property. The Δ/V_N term was referred to as the Delta Ratio.

Special Cases

As with the derivation of the original model by Elfar, there were a number of special cases that occurred when T and/or i assumed certain values. These special cases occurred when

$T = 1$ and $i > 0$

$T = \infty$ and $i > 0$

$T = 1$ and $i = 0$

$T = \infty$ and $i = 0$

$T < \infty$ and $i = 0$

For each of these cases, the general form of the equation became indeterminant. Therefore, a derivation of the equation for each of the special cases was needed.

<u>T = 1 and i > 0</u> As presented earlier, the T = 1 case represented a situation in which the operation returns declined each interval by a constant amount. The decline each period was equal to (R_1/N). The operation returns for any interval (x-1) to x, then were:

$$R_x = R_1 - (x-1)(R_1/N)$$

$$= R_1\left[\frac{N-x+1}{N}\right] \qquad (35)$$

Referring to Elfar's dissertation (14, p. 57), the expression for R_1 when T = 1 was shown to be:

$$R_1 = \frac{V_N(1-S(p/f)^i_N)}{(p/a)^i_N - (1/N)(p/g)^i_N} \qquad (36)$$

Using Equation 23 as the starting point, then substituting Equations 35 and 36 into Equation 24, resulted in the following:

$$\Delta = R_1 - R_x \qquad (23)$$

$$= R_1 - R_1\left[\frac{N-x-1}{N}\right]$$

$$= R_1 \frac{x-1}{N}$$

$$\Delta = \frac{V_N\left[1-S(p/f)^i_N\right]\left[x-1\right]}{\left[(p/a)^i_N - (1/N)(p/g)^i_N\right]\left[N\right]} \qquad (37)$$

Dividing both sides by V_N resulted in:

$$\frac{\Delta}{V_N} = \frac{(1-S(p/f)^{i}_{N})(x-1)}{(N(p/a)^{i}_{N}-(p/g)^{i}_{N})}$$

$$\frac{\Delta}{V_N} = \frac{(i^2)((1+i)-S)(x-1)}{(1+i)^N(Ni-1) + 1} \tag{38}$$

This equation was the final equation for the special case of T = 1.

<u>T = ∞ and i > 0</u> Recalling Equation 7, applying L'Hospital's rule by differentiating the numerator and denominator, then finding the limit as $T \rightarrow \infty$, resulted in the following:

$$R_x = R_1 \left[\frac{T^N - T^{x-1}}{T^N - 1}\right] \tag{7}$$

$$R_x = \lim_{T \to \infty} R_1 \left[\frac{NT^{N-1} - (x-1)T^{x-2}}{NT^{N-1}}\right]$$

$$R_x = \lim_{T \to \infty} R_1 \left[1 - \frac{(x-1)}{(N)} \frac{1}{(T^{N-x+1})}\right]$$

$$R_x = R_1 \tag{39}$$

Since $R_1 = R_x$ for this case, Equation 23 resulted in the following:

$$\Delta = R_1 - R_x \tag{23}$$

$$= R_1 - R_1$$

$$\Delta = 0 \tag{40}$$

Since T = ∞ indicated that the operation returns were uniform from interval to interval, one would expect that the Δ value would be zero in magnitude. Equation 40, then, was the resulting expression for the T = ∞ special case.

<u>T = 1 and i = 0</u> This case represented the situation of a uniform decline in operation returns under a zero discount rate situation. Since

T = 1, Equation 35 again applied for the R_1 term. Also, once again taking advantage of Elfar's previous derivations (14, p. 64), R_1 was shown, for this case, to be:

$$R_1 = \frac{2V_N(1-S)}{(N+1)} \quad (41)$$

Combining equations 23, 35, and 41 resulted in the following:

$$\Delta = R_1 - R_x \quad (23)$$

$$= R_1 - R_1\left[\frac{N-x+1}{N}\right]$$

$$= R_1\left[\frac{x-1}{N}\right]$$

$$\Delta = \left[\frac{2V_N(1-S)}{(N+1)}\right]\left[\frac{x-1}{N}\right] \quad (42)$$

$$\frac{\Delta}{V_N} = \frac{2(1-S)(x-1)}{N(N+1)} \quad (43)$$

This equation was the final expression for the T = 1 and i = 0 special case.

T = ∞ and i = 0 Since the T = ∞ represented the uniform operation returns situation regardless of what the discount rate was, the final equation was the same as the previous T = ∞ case. That is,

$$\frac{\Delta}{V_N} = 0 \quad (40)$$

still applied in this special case also.

T < ∞ and i = 0 For this case, Equations 23 and 7 were again the best starting point. Therefore,

$$\Delta = R_1 - R_x = R_1 - R_1\left[\frac{T^N - T^{x-1}}{T^N - 1}\right] \quad (23)$$

$$\Delta = R_1\left[\frac{T^{x-1} - 1}{T^N - 1}\right] \quad (44)$$

Using Equation 25 and a value of i = 0 resulted in the following:

$$R_1 = \frac{V_N(1-S(p/f)_N^i)(T^N-1)}{\sum_{m=1}^{N}(T^N-T^{m-1})(p/f)_m^i} \tag{25}$$

$$= \frac{V_N(1-S)(T^N-1)}{\sum_{m=1}^{N}(T^N-T^{m-1})}$$

$$= \frac{V_N(1-S)(T^N-1)}{NT^N-T^{-1}\sum_{m=1}^{N}T^m}$$

$$= \frac{V_N(1-S)(T^N-1)}{NT^N-T^{-1}\left[\frac{T(T^N-1)}{T-1}\right]}$$

$$R_1 = \frac{V_N(1-S)(T^N-1)(T-1)}{T^N(NT-N-1)+1} \tag{45}$$

Substituting Equation 45 into Equation 44, and dividing both sides by V_N, resulted in the following expression:

$$\frac{\Delta}{V_N} = \left[\frac{(1-S)(T^N-1)(T-1)}{T^N(NT-N-1)+1}\right]\left[\frac{T^{x-1}-1}{T^N-1}\right] \tag{46}$$

$$\frac{\Delta}{V_N} = \frac{(1-S)(T-1)(T^{x-1}-1)}{T^N(NT-N-1)+1} \tag{47}$$

This equation was the final expression for the special case of i = 0.

Summary of Equations

A summary of the equations derived for this section is as follows:

General Form

$$\frac{\Delta}{V_N} = \frac{(q^N-S)(T^{x-1}-1)(T-q)(i)}{T^N(Tq^N-T-q^{N+1}+1)+(iq^N)} \tag{34}$$

T = 1 and i > 0

$$\frac{\Delta}{V_N} = \frac{(1-S(p/f)^{i}_{N})(x-1)}{(N(p/a)^{i}_{N}-(p/g)^{i}_{N})} = \frac{(i^2)((1+i)^N-S)(x-1)}{(1+i)^N(Ni-1)+1} \qquad (38)$$

T = ∞ and i > 0

$$\frac{\Delta}{V_N} = 0 \qquad (40)$$

T = 1 and i = 0

$$\frac{\Delta}{V_N} = \frac{2(1-S)(x-1)}{N(N+1)} \qquad (43)$$

T = ∞ and i = 0

$$\Delta = 0 \qquad (40)$$

T < ∞ and i = 0

$$\frac{\Delta}{V_N} = \frac{(1-S)(T-1)(T^{x-1}-1)}{T^N(NT-N-1)+1} \qquad (47)$$

These equations were used to estimate a value of T, the progression rate, for the property in question. Knowing a value for T, the value at any age x could then be computed using Elfar's model.

Procedural Steps for the Delta Procedure

The proposed procedure for estimating T-factor values has several steps. These steps, and a brief explanation of each, are summarized below:

Quantify P (Periodic Reduction in Operation Returns) The most readily available data concerning the reduction in operation returns are the repair and maintenance records compiled by most companies. Downtime losses are often not recorded as a part of normal cost accounting procedures, so they may have to be estimated from incomplete records. Estimation of the downtime component of depreciation is often simplified, however, by its tendency to be so small that it is considered to be negligible. Production rate loss, if it can be shown to be a result of age, can also be easily computed if sufficient cost accounting records are available. If not, then it also will have to be estimated from incomplete records. The final major component of P is obsolescence. Though this component is rarely recorded as a part of a cost accounting system, it can be estimated using the Affanasiev procedure presented earlier. When the major components of P for each period have been computed, measured, or estimated, adjusted to units of constant dollars, and expressed in units of dollars per period, they can then be summed to determine the total periodic amount of the reduction in operation returns.

Compute Δ The Δ term is the numerical difference between P_1 and P_x where P_x is the reduction in operation returns for the period x-1 to x. The computation of Δ is accomplished by subtracting the value of P_1 from successive values of P_x determined in the previous step.

Determine V_N The V_N term is the value of the property when it was new. Normally, the original cost, trended or untrended as appropriate, is the best indication of the value new term. In some cases, however, the current reproduction cost of a new unit may be easier to find and just as correct to use.

Compute Δ/V_N Computation of the Delta Ratio (Δ/V_N) is performed for each successive period by dividing the V_N term determined in the previous step into the Δ values computed in an earlier step.

Determine the Probable Service Life The probable service life is determined using the Iowa type curves, or other comparable procedure, if sufficient life analysis data are available. If not, then the probable service life must be estimated based on the best information available. Reliability of the results will, as in all cases where estimated values are used, depend on the accuracy of the estimated value.

Estimate the Value of T Using a Set of Standard Curves Since solution of Equation 34, or any of its special case equations, for the T-factor value is at best a trial and error procedure, a procedure based on the visual matching of observed data to theoretical, calculated results will be used to estimate T-factor values. Applications of the proposed procedure to experimental data will be presented in a later section.

Group Property Considerations

The terminology and procedures previously presented are either based on or strongly biased toward unit properties. Though applying the proposed procedure to group properties greatly complicates the calculations, the basis of the procedure and validity of the results are unchanged. The significant differences in applying the proposed procedure, however, should be explained.

Explanation of Procedure Differences   The application of the Delta Procedure to group property accounts is based on dividing the group property into frequency groups having equal probable lives. This separation and collection of frequency groups is a function of the Iowa type curve exhibited by the group property. Each frequency group is then treated as a unit property in the application of the equations and procedures previously derived.

Using the T-factor values estimated for the frequency groups, a modified condition percent factor is then calculated for each group. Multiplying each of these modified condition percent factors by the proportion of units contained in each frequency group, then summing the products will give a weighted modified condition percent factor for the entire group property. If an age other than zero is used, only the frequency groups surviving beyond the age selected are used in the computation.

Example Calculation   To complete the group property discussion, an example is presented in Table 2 to demonstrate application of the proposed procedure. Values of the T-factor and surviving dollars for each frequency group are assumed for illustrative purposes only. Application to an actual group property account may vary slightly if the available data were not in exactly the same format.

Table 2. Example of value estimation calculation for a group property

Age	Surviving dollars	Estimated T-factor	Modified condition percent factor	Estimated value
0	0		1.000000	0
1	50,000	.80	.639760	31,988
2	100,000	.83	.473267	47,327
3	60,000	.85	.373547	22,413
4	130,000	.90	.406225	52,809
5	0	.97	.540517	0
6	50,000	.94	.374118	18,706
7	50,000	.90	.198094	9,905
8	30,000	.95	.283473	8,504
9	70,000	.87	.071205	4,984
10	120,000	.86	.042184	5,062
11	0	.84	.018742	0
12	40,000	.93	.085461	3,418
13	90,000	.82	.004690	422
14	50,000	.89	.019354	968
15	60,000	.92	.025029	1,502
16	40,000	.98	.046266	1,851
17	50,000	.96	.019646	982
18	10,000	.84	.000395	4
19	0	.94	.001841	0
	$1,000,000		Total estimated value =	$210,845

EXPERIMENTAL PROCEDURES

Two sets of data were sought for this study. The first included market evidence data, depreciation data, and data indicating probable service lives and value new for several select equipment types. Since a T-factor value could be estimated using techniques other than the proposed Delta or Ratio Procedures, this data set was used both to validate the T-factor model and as a control to test the proposed procedure against. The second set included depreciation, value new, and probable service life data for a number of equipment types owned by several Iowa industries. Since an insufficient amount of data were available to apply the Ratio Procedure, only the Delta Procedure was used to estimate T-factor values for these properties. These T-factor values were then used to calculate the cost evidences of value at any age.

Estimation of T for a Known Property

Prior to testing the proposed Delta Procedure, two tasks had to be accomplished. First, the T-factor model had to be validated, and second, a control had to be established to test the proposed procedure against. Both of these tasks were accomplished by collecting data for several select equipment types that had a strong and known market evidence curve. This curve gave a value vs. time relationship that could be used to find a T value using appropriate curve-fitting techniques.

Data Collected

Data for the following six different equipment types with known market evidence curves were sought:

1. Caterpillar D8 dozers
2. 3/4 ton pickup trucks
3. Caterpillar 651 self-propelled scrapers
4. large production, single engine, small aircraft
5. Boeing 707 aircraft
6. industrial forklifts

The lack of suitable data resulted in three of the equipment types (scrapers and both aircraft types) being discarded during the data collection stage. The other three equipment types (dozers, pickups, and forklifts) were fully developed and used to estimate T-factor values.

Caterpillar D8 Dozers Data for D8 dozers were sought for several reasons. First, information appeared to be readily available. Data concerning original cost, estimated salvage values, and probable service lives could be obtained from equipment dealers. Data concerning the rate of return, depreciation factors, and operational characteristics, on the other hand, were routinely recorded and analyzed by construction contractors. Finally, data resulting in a market evidence curve were available from equipment auction firms.

A second reason for the selection of D8 dozers was the similarity of characteristics between construction and industrial equipment. Preliminary data collection revealed that downtime was minimized at the expense of repair and maintenance costs, that the basis of value was the machine's ability to produce, and that the operation returns generally decreased with age (22).

A third reason for pursuing data about D8 dozers was that an extensive resale market for used equipment existed. Because D8 dozers had not undergone rapid technological advances in the last 20 years, the resale market gave a strong market evidence of value at any age. The large number of sales in a given year improved the reliability of this market evidence curve.

To apply Elfar's model, the data collected had to be at least extensive enough to determine a value for each parameter contained in the model, with one exception. That exception was T, the progression rate. Refering to Equations 9 and 10, these parameters included the following:

V_N - value when new

V_S - net salvage value

V_x - value at age x

N - probable service life

r - rate of return

x - age of equipment

With known values for these parameters, a value of T could be estimated.

The value when new was easily determined for most properties, particularly when it was set equal to the original cost. This original cost should include the total purchase price, any transportation or freight charges, and all installation costs. For the D8 dozers, the original cost of an average unit in central Iowa was $165,000.00 in 1978 (23). The 1978 cost was used so that the V_N value would correspond to the market evidence curve data collected. As a result, the original cost, trended original cost, replacement cost, and reproduction cost definitions were all equivalent in magnitude.

The salvage value was strongly evidenced by the eventual net proceeds received from the sale of the property upon retirement. Net proceeds were equal to gross salvage price less any costs of removing the property. Since the actual salvage value was unknown until the actual point of retirement, an estimated value was used. For the D8 dozer account, the salvage value was estimated using the market evidence curve. That is, the salvage value was set equal to the value of the market evidence curve at the computed probable life value. Because of the almost negligible magnitude of the market evidence curve beyond 32 years of age, salvage values at probable life values of greater than 32 years were set equal to zero.

The value at age x was judgementally determined based on the market evidence curve. The best source of market evidence data for D8 dozers was found to be an auction summary published annually by a large midwestern equipment auction dealer (24). A summary of these data for 1978 is shown in Table 3, and a plot of the market evidence curve is shown in Figure 4. Since all of the values shown in Figure 4 are in 1978 dollars, no adjustment of the curve for time is necessary.

The probable service life was another quantity that was not known until the actual point of retirement. Based on lifespans of similar units and various life analysis techniques, however, the probable service life can often be estimated with a reasonable degree of accuracy. Unfortunately, no life analysis studies for D8 dozers could be found. Therefore, the probable service life for D8 dozers was estimated, based

Table 3. Market evidence data for D8 dozers

Unit No.	Model year	Month sold	Age (mons.)	Sale price
1	1977	JUN	11	110,000
2	1977	APR	9	107,000
3	1976	APR	21	96,000
4	1975	APR	33	87,500
5	1975	APR	33	77,500
6	1975	JUN	35	104,500
7	1974	JUN	47	99,000
8	1974	JUN	47	94,500
9	1973	APR	57	63,000
10	1973	MAY	58	65,000
11	1973	MAY	58	73,000
12	1972	APR	69	58,000
13	1972	APR	69	52,000
14	1972	APR	69	53,000
15	1969	APR	105	41,000
16	1968	JUN	119	47,500
17	1967	JUN	131	43,000
18	1967	MAY	130	35,000
19	1966	APR	141	34,000
20	1965	JAN	150	16,500
21	1964	JUN	167	21,000
22	1964	JUN	167	22,000
23	1964	MAY	166	37,500
24	1963	APR	177	23,000
25	1963	MAY	178	14,000
26	1963	MAR	176	19,500
27	1961	APR	201	27,000
28	1960	JAN	210	12,250
29	1960	MAY	214	19,000
30	1963	JAN	174	13,000
31	1961	MAY	202	16,500
32	1958	APR	237	8,000
33	1958	APR	237	11,750
34	1957	FEB	247	11,250
35	1956	APR	261	9,000
36	1956	MAY	262	12,500
37	1956	MAY	262	12,500
38	1955	MAY	274	9,200
39	1950	FEB	331	3,500
40	1975	NOV	41	73,000

Table 3. (continued)

Unit No.	Model year	Month sold	Age (mons.)	Sale price
41	1975	DEC	42	92,500
42	1975	JUL	37	78,000
43	1975	JUL	37	76,000
44	1975	SEP	39	105,000
45	1975	OCT	40	88,000
46	1975	JUL	37	78,000
47	1974	JUL	49	72,000
48	1974	OCT	52	52,500
49	1974	DEC	54	60,000
50	1974	DEC	54	64,000
51	1973	NOV	65	55,000
52	1973	OCT	64	50,000
53	1973	JUL	61	50,000
54	1973	JUL	61	50,500
55	1973	JUL	61	49,500
56	1972	DEC	78	58,000
57	1972	DEC	78	52,500
58	1971	NOV	89	39,000
59	1970	OCT	100	34,500
60	1969	JUL	109	45,000
61	1969	AUG	110	39,000
62	1969	JUL	109	40,000
63	1969	DEC	114	45,000
64	1969	AUG	110	24,500
65	1969	DEC	126	29,000
66	1968	AUG	122	29,750
67	1968	DEC	126	27,000
68	1968	AUG	122	27,500
69	1968	OCT	124	46,000
70	1967	JUL	133	31,000
71	1967	JUL	133	26,000
72	1967	DEC	138	33,500
73	1965	JUL	157	22,500
74	1965	DEC	162	19,250
75	1964	AUG	170	19,500
76	1964	AUG	170	14,500
77	1963	OCT	184	15,750
78	1963	DEC	186	14,750
79	1963	DEC	186	18,500
80	1963	DEC	186	15,250

Table 3. (continued)

Unit No.	Model year	Month sold	Age (mons.)	Sale price
81	1963	OCT	184	9,700
82	1962	OCT	196	13,000
83	1962	DEC	198	14,750
84	1962	OCT	196	10,500
85	1962	OCT	196	13,750
86	1961	AUG	206	29,000
87	1961	OCT	208	13,600
88	1961	AUG	206	12,750
89	1961	OCT	208	9,500
90	1960	OCT	220	14,000
91	1960	DEC	222	12,000
92	1959	SEP	231	18,000
93	1959	AUG	230	22,000
94	1959	JUL	229	25,800
95	1959	JUL	229	21,500
96	1959	JUL	229	20,000
97	1959	JUL	229	23,250
98	1959	AUG	230	10,750
99	1956	OCT	268	15,000
100	1956	SEP	267	13,000
101	1953	AUG	302	7,700
102	1948	JUL	361	2,750

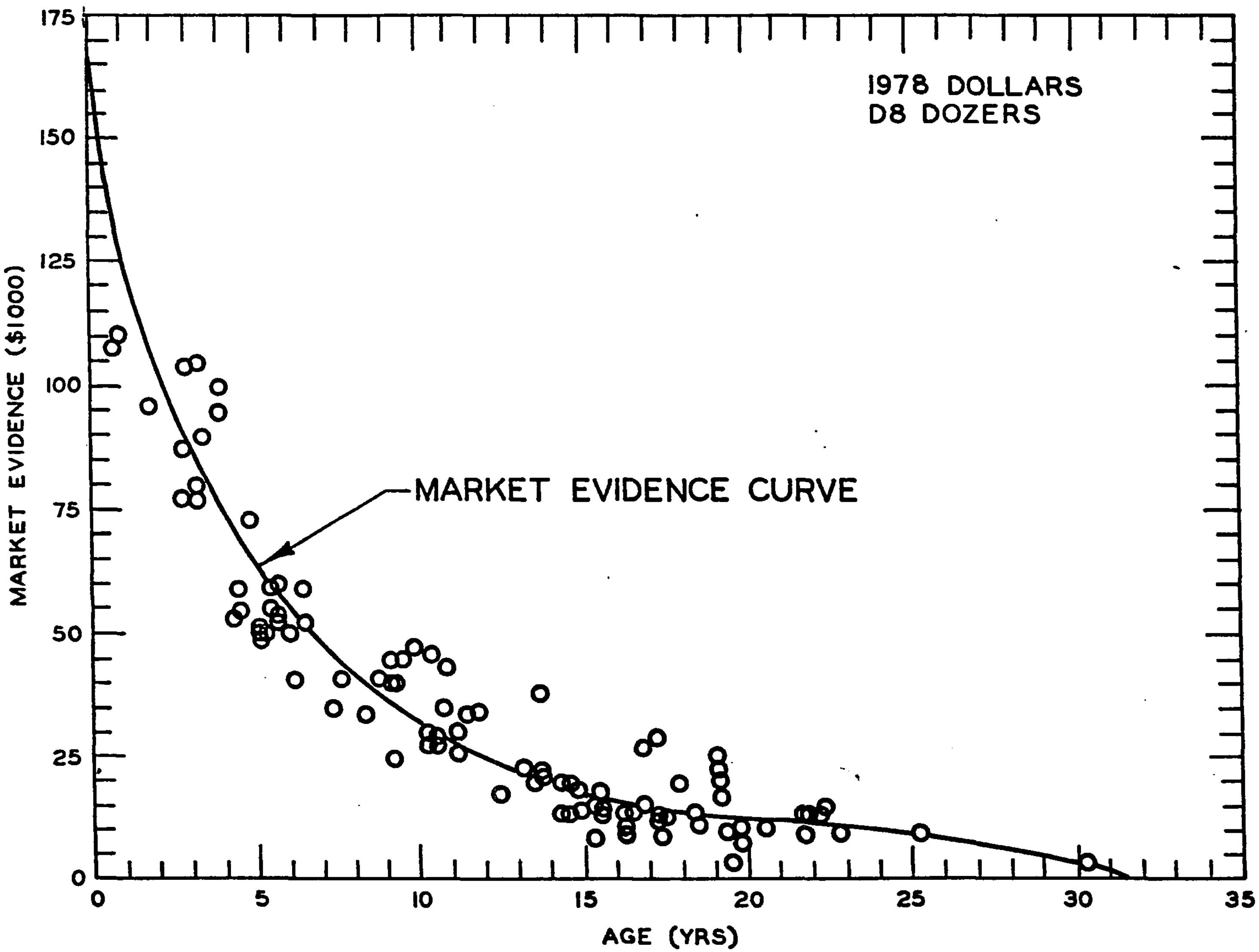

Fig. 4. Market evidence data for D8 dozers

on industry experience (22), to be about 20 years.

In connection with the determination of the probable service life, another piece of data, the appropriate Iowa type curve, also had to be found. Though the D8 dozers were handled as a unit property, the Iowa type curves were still needed to compute the probable life values for any age other than zero. As the unit ages, the probable life increases. Selection of the curve most reflective of this property was done through a trial and error procedure using a digital computer to match the computed value curve to the market evidence data for various values of T and selections of Iowa type curves.

Although the rate of return, or discount rate, could be selected to reflect the company's financial policy and economic conditions prevalent at the time of the valuation, such a selection would be erroneous. Rather, the discount rate should be chosen to reflect a reasonable, but inflation-free rate of return for the company. Several excellent arguments can be advanced supporting the use of an inflation-free rate of return. One of these is that since the rate of inflation tends not to be a constant amount, the selection of a single discount rate is quite inaccurate. Another argument involves the mathematical inaccuracies that result when the modified condition percent is computed based on an inflation discount rate and placed in the same equation with a value new based on uninflated dollars. The mixing of dissimilar dollar units leads to an inaccurate calculation of the value at age x. Another argument was advance by Elfar when he stated that the calculated operation return stream is in constant dollars and should therefore be discounted using an inflation-free rate (14, p. 120). Based on these arguments, it is strongly recommended that an inflation-free rate of return be used. Severe inflation can be included as another component of value depreciation, or eliminated as a factor by using indices to convert all costs into units of constant dollars. As a reasonable approximation of an inflation-free rate of return, an annual rate of r = 7% was used for D8 dozers (25).

The age of the property is a parameter that is either specified or left as an incrementing variable depending on whether value at a specific age or value at any age is desired. For the D8 dozers, value at any age was the desired end product.

<u>3/4 Ton Pickup Trucks</u> Data were also collected for 3/4 ton pickup trucks used as construction vehicles. In addition to the readily available information, similarity with industrial equipment, and extensive resale market reasons cited for the D8 dozer property, the pickup property also had a stub survivor curve available for life analysis purposes. Data were collected to determine the same parameters as were identified in the D8 dozer discussion.

Determination of the value when new for the pickup property was complicated by the fact that the original cost probably contained some "new paint" value. New paint value may be defined as the excess value a new property item has over and above the value evidenced by future usefulness. It is normally the result of an increased desirability of ownership occasioned solely by the brand new status of the property. Existence of a "new paint" component of value was recognized and briefly discussed as long ago as 1916 by Campbell (11, p. 14). Since the new trucks are commonly altered be adding a number of accessories to the stock model to ready it for construction use, the assumption was made that the value of the accessories approximately offset the new paint value of the truck.

Two reliable sources were identified and used for collection of value new data. One was a large, midwestern heavy construction contractor that maintained quite extensive and very reliable cost records (22). The other source was the National Automobile Dealer's Association (NADA) published prices for used pickups (26). According to these two sources, the average original cost for a 3/4 ton pickup, in 1978 dollars, was $6474.00 (contractor records) or $5450.00 (NADA). Since neither source could be verified as definitely superior to the other, both evidences of the value new were carried forth in the calculations.

The salvage value for the pickup property was estimated in the same manner as for the D8 dozer property. That is, the salvage value was set equal to the market evidence curve value at the computed probable life. Beyond that point where the market evidence curve intersected the horizontal axis, the salvage value was set equal to zero.

The value at age x was again determined to be best evidenced by the market evidence curve. As with the original cost, two sources of data were identified and used. These two sources were the NADA published prices (26) and the construction contractor's "current replacement" values (22). A tabular presentation and graphical plot of the market evidence data are shown in Table 4 and Figure 5 respectively for both

Table 4. Market evidence data for 3/4 ton pickup trucks

Model year	Age (yrs)	Estimated value (NADA)	Estimated value (contractor)
1979	0	5450	6474
1978	1	4383	3496
1977	2	3667	2590
1976	3	2967	1619
1975	4	2517	1295
1974	5	1933	971
1973	6	1550	777
1972	7	1225	518
1971	8		388
1970	9		324
1969	10		259

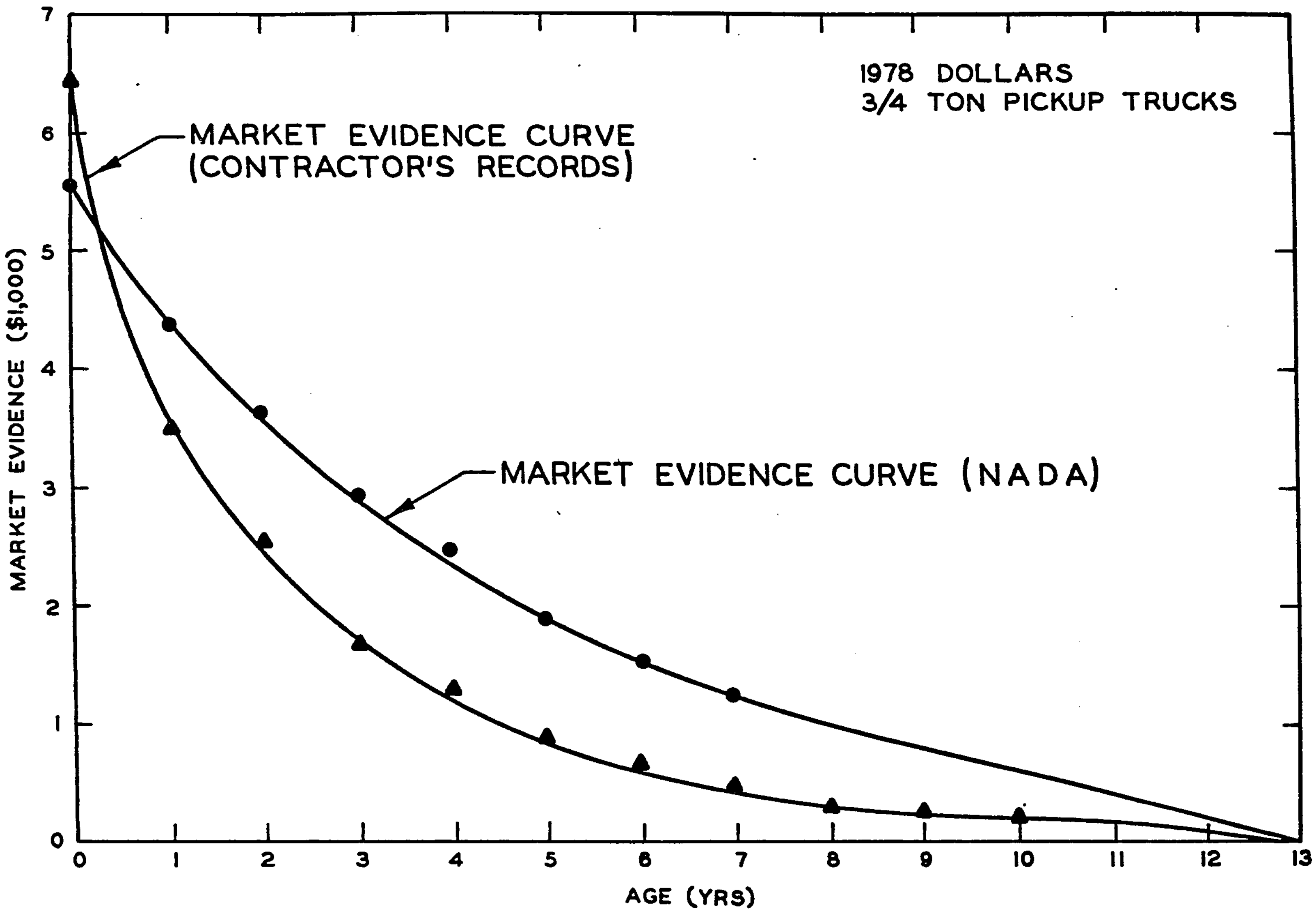

Fig. 5. Market evidence data for pickup trucks

sources.

Estimation of a probable service life for the 3/4 ton pickup property was greatly facilitated by the availability of some life analysis data (27). Based on this data source, a stub survivor curve, shown in Figure 6, was plotted. The Iowa type curve selected using this stub curve was used to estimate the probable service life and calculate the probable life for salvage value determination.

The rate of return and age of the equipment were handled in the same manner as for the D8 dozer property. That is, an annual rate of 7% was used (25), and the age was incremented to obtain value at any age x.

Industrial Forklifts To avoid an unintentional bias caused by using exclusively construction equipment types, data were also sought for industrial forklifts. The primary source of data for this property group was the rental division of a large forklift manufacturer. A fairly typical forklift model currently in widespread use was selected for data collection.

The value new was estimated to be $18,350.00 in 1979 dollars. This figure was based on the average retail price supplied by the forklift manufacturer.

Estimated salvage values were based, as with the D8 dozer and 3/4 ton pickup properties, on the market evidence curve. The best evidence of salvage value was the market evidence curve amount at the computed probable life.

The value at age x, as before, was best estimated using the market evidence curve. Data for this curve were obtained from a national

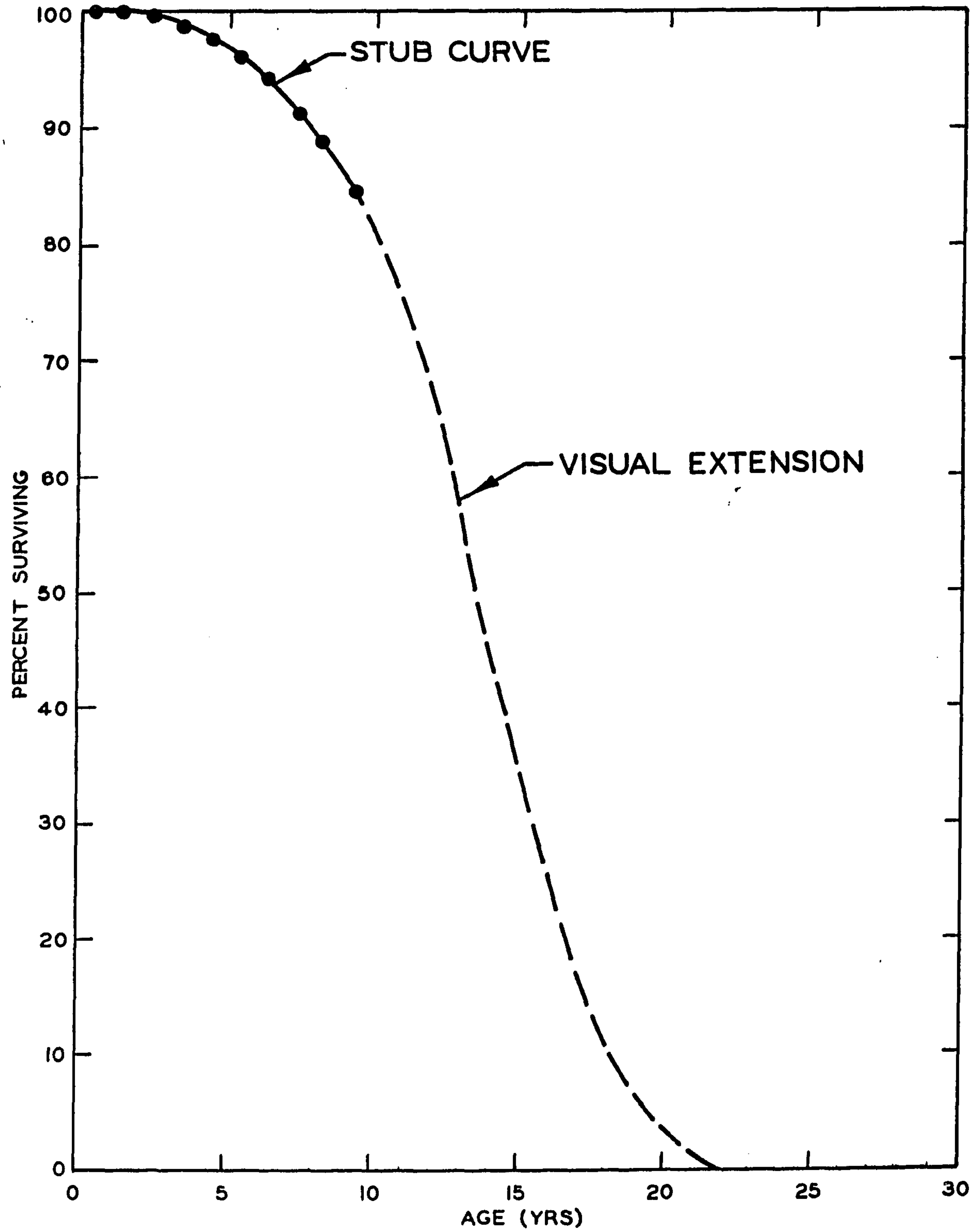

Fig. 6. Stub survivor curve for pickup trucks

publication specializing in the compilation of industrial equipment resale prices (28). A graphical plot of the market evidence curve data is shown in Figure 7. Since the used equipment price data were available for only a 10 year span and since a sudden drop in the market evidence data to zero would cause a large disruption to the curve fitting methods used, the market evidence curve was visually extended until it intersected the horizontal axis at about 20 years of age. Though no evidence was available supporting the extension of the market evidence curve, it was felt that the error in estimating value at any age x would be less with the extended curve than if a curve suddenly plunging to zero at 10 years of age were used. Salvage values beyond 20 years of age were set equal to zero.

The determination of probable life was made using a procedure similar to the one used for the D8 dozer property. Since no reliable life analysis data were available, a probable service life of 10 years was estimated based on discussions with experienced industry sources (29). Selection of the most appropriate Iowa type curve was based on a trial and error fitting procedure first developed for the D8 dozer property. The Iowa type curve thus selected was used in the salvage value determination.

The rate of return was again set equal to the 7% inflation-free value (25) previously selected for the D8 dozer and 3/4 ton pickup properties. Also, the equipment age was again incremented to give value at any age.

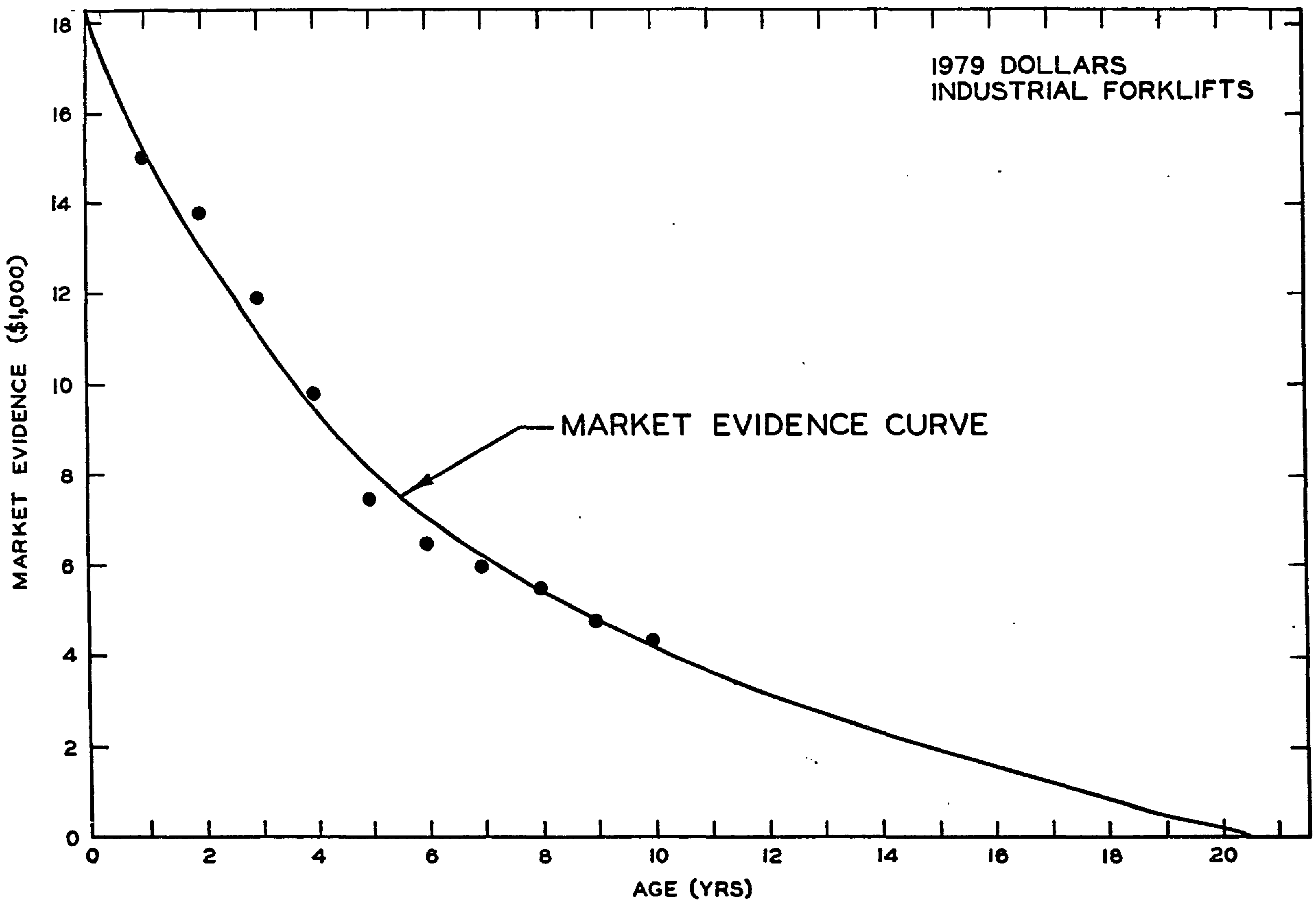

Fig. 7. Market evidence data for industrial forklifts

Experimental Procedure Followed

Upon completion of the data collection for this stage, T-factor values were estimated for the D8 dozer, 3/4 ton pickup, and industrial forklift properties. The procedure followed was to compute several value curves by varying the T-factor value and using the Elfar model. The computed value curves were then compared to the corresponding market evidence data using a sum-of-the-square-of-the-differences procedure to measure the goodness of fit. The T-factor value resulting in the best fit was then selected as being the progression rate exhibited by the property. Results of this procedure are presented in a later section.

Estimation of T Using the Delta Procedure

Data were also sought for properties that would allow the tested and untested application of the proposed Delta Procedure. The tested application was performed using the D8 dozer, 3/4 ton pickup, and industrial forklift properties previously discussed. Additional data evidencing the rate of depreciation accrual were collected prior to applying the proposed Delta Procedure. Data were also collected for several Iowa industrial properties so that an untested application of the proposed Delta Procedure could be made.

Additional Data Collected for Properties with Known Market Evidence Data

Estimation of T-factor values based on market evidence data was accomplished using the information and procedures described in the last subsection. Estimation of T-factor values using the proposed Delta Procedure, however, required collection of additional data related to the components of depreciation. The normally increasing components of depreciation commonly include one or more of the following:

1) rising repair and maintenance costs as parts wear out or fail in service
2) decreasing availability for service due to increasing downtime
3) falling production rates
4) increasing functional or economic obsolescence.

Therefore, data showing the occurrence and magnitude of these depreciation components for D8 dozers, 3/4 ton pickups, and industrial forklifts were collected.

D8 Dozers Data collected for the D8 dozer account indicated that substantial changes or improvements have not been made during the last 20 years (22). Therefore, functional obsolescence was considered to be negligible. In addition, the D8 dozer property suffered no significant loss in hourly production rates with age (22). Finally, the property exhibited a 99.5% availability (22), inferring that downtime was also negligible. The major cause of depreciation for the D8 dozer property, therefore, was found to be the increasing levels of repair and maintenance expenditures. A tabular summary of the repair and maintenance data is shown in Table 5. The data contained in Table 5 were derived from field reports of time and costs received by the contractor's home office. A regression analysis and cost index were applied to the field data by the contractor during the summarization process. The results were then plotted as constant dollar, smoothed curves for three subcategories of repair and maintenance. Data from these three plots

Table 5. Summary of repair and maintenance expenses for D8 dozers

Machine hours	Total cost per machine hour ($/hr)
500	15.55
1000	14.68
1500	14.85
2000	14.82
2500	15.06
3000	15.21
3500	15.54
4000	15.87
4500	16.38
5000	16.42
5500	16.78
6000	17.05
6500	17.04
7000	17.31
7500	17.46
8000	17.89
8500	18.31
9000	18.69
9500	18.92
10000	19.27

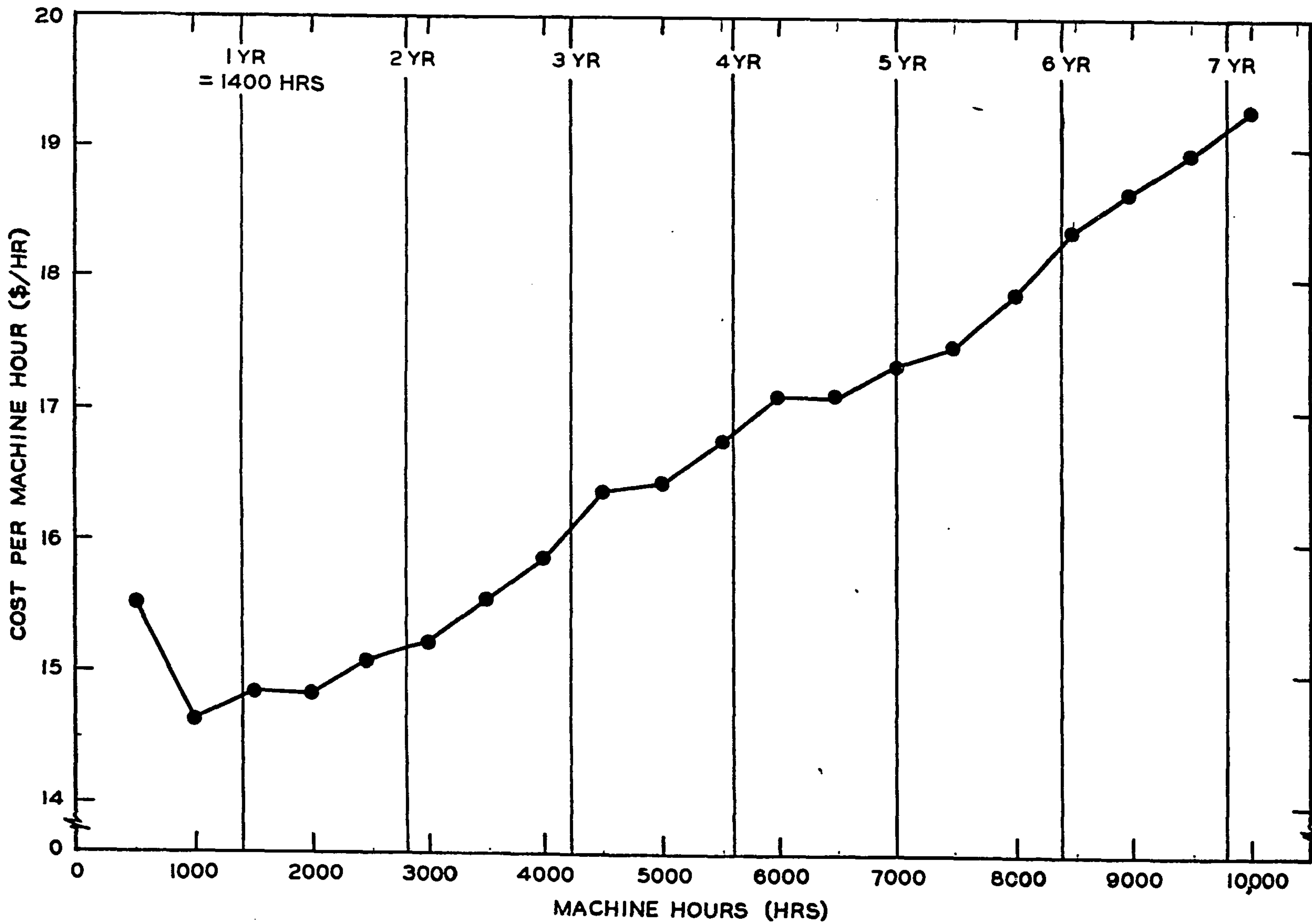

Fig. 8. Repair and maintenance costs for D8 dozers

were then combined to produce the information contained in Table 5. A graphical plot of this data is shown in Figure 8.

<u>3/4 Ton Pickups</u> Data collected for the 3/4 ton pickup property indicated that the downtime production rate loss, and obsolescence components of depreciation were negligible. Data concerning repair and maintenance costs were obtained from the same source as for the D8 dozers (22). Therefore, the same field collection, regression analysis, indexing, and summarization procedures were followed in the compilation of the final data presented in Table 6. A graphical plot of these data is shown in Figure 9.

The plot shown in Figure 9 exhibits a distinctive discontinuity in its data. The reason for the increase-dramatic drop-increase pattern shown was traced to a shift of primary utilization and responsibility during the company's ownership. These two stages resulted from the pickup entering service assigned to a specific person, then being transferred to "job truck" status at a later date. This second stage did not actually decrease repair and maintenance costs as indicated in Figure 9. Rather, since everybody's truck became nobody's truck, routine, inexpensive maintenance items were deferred until they became mandatory, expensive repairs. Though this discontinuity in the depreciation data affected the accuracy of the T-factor estimation, it was still possible to apply the proposed Delta Procedure.

<u>Industrial Forklifts</u> Data collected concerning depreciation for the industrial forklifts indicated that the obsolescence and production rate

Table 6. Summary of repair and maintenance expenses for pickup trucks

Machine hours	Total cost per machine hour ($/hr)
500	.66
1000	.56
1500	.53
2000	.52
2500	.55
3000	.57
3500	.60
4000	.64
4500	.65
5000	.66
5500	.65
6000	.66
6500	.67
7000	.65
7500	.57
8000	.52
8500	.46
9000	.47
9500	.47
10000	.53

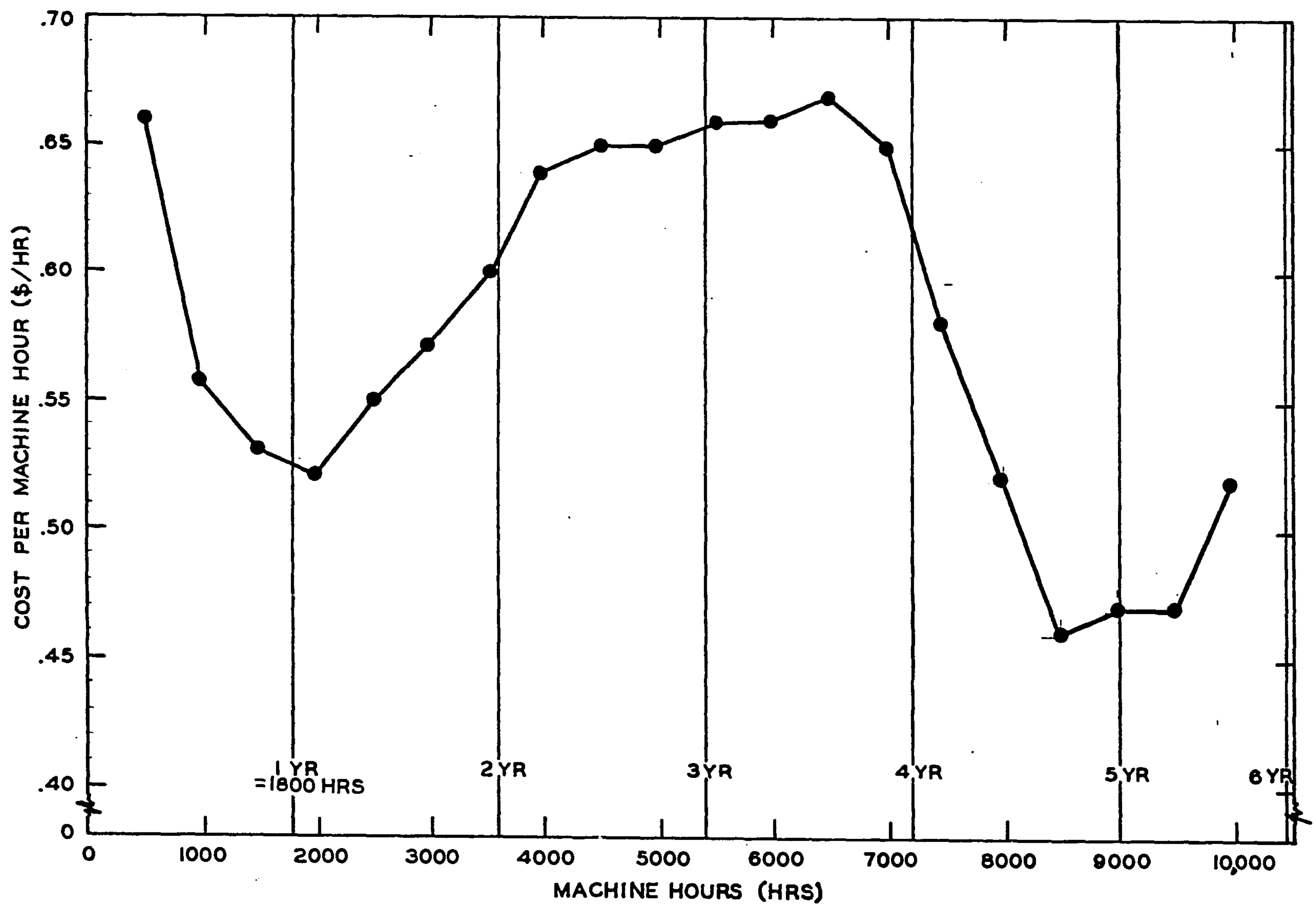

Fig. 9. Repair and maintenance costs for pickup trucks

loss components were negligible. Recorded costs for downtime and repair and maintenance expenses were made available in an unpublished report supplied by a forklift rental firm (29). A tabular summary and graphical plot of these data are shown in Table 7 and Figure 10 respectively.

<u>Data Collected for Properties with Unknown Market Evidence Data</u>

Data were collected for several industrial equipment properties owned by Iowa firms. A brief listing of these properties as as follows:

1) Property A - 16 die cast machines
2) Property B - 17 trim presses
3) Property C - 22 printing presses
4) Property D - 26 platform trucks
5) Property E - 3 forklift trucks

The lack of market evidence data made these five properties more representative of typical industrial valuation situations than the properties previously discussed. In addition, the five property groups listed above were representative of the equipment owned by the Iowa industrial firms sponsoring this study. Therefore, data were collected from these properties so that the proposed Delta Procedure could be applied.

<u>Property A</u> Data were available for each unit of Property A that gave the year of installation and the original cost. In addition, company records provided an appropriate cost index to adjust all dollar figures to a constant dollar basis. Company personnel knowledgeable about this property estimated the probable service life to be about 30 years. Data

Table 7. Summary of repair, maintenance, and downtime expenses for industrial forklifts

Machine hours	Repair and maintenance cost ($/hr)	Downtime cost ($/hr)	Total cost ($/hr)
2,000	.25	.12	.37
4,000	.32	.16	.48
6,000	.36	.18	.54
8,000	.47	.23	.70
10,000	.55	.28	.83
12,000	.58	.29	.87
14,000	.63	.31	.94
16,000	.69	.35	1.04
18,000	.73	.37	1.10

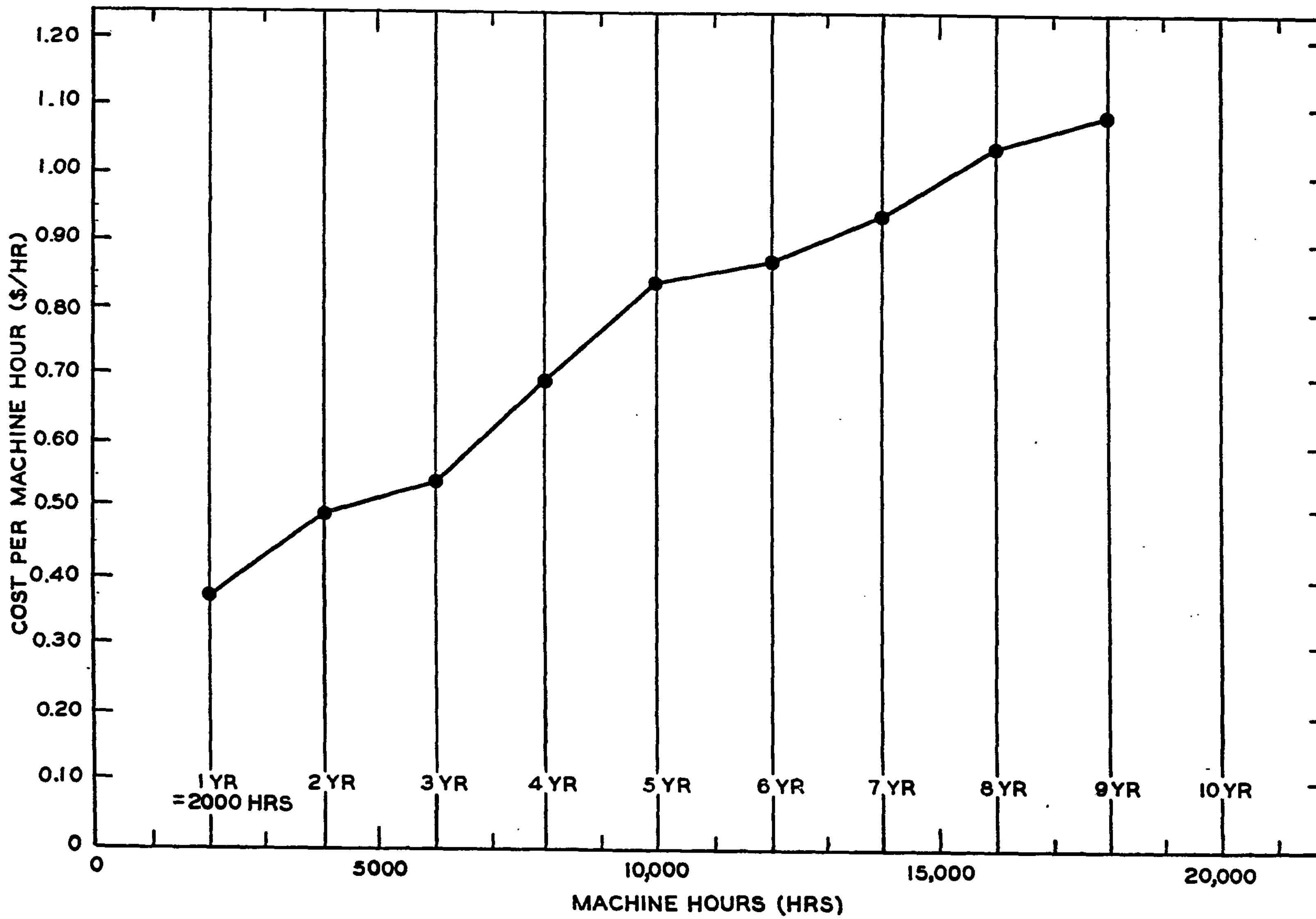

Fig. 10. Repair, maintenance, and downtime costs for forklifts

reflecting repair and maintenance costs for three to 20 years, depending on the age of the unit, were also available. A tabular summary of the repair and maintenance data, in constant dollars, is given in Table 8. A plot of these data is shown in Figure 11. Downtime costs were negligible since the company chose to incur premium rate, offshift repair and maintenance costs rather than have a machine unavailable during regular operating hours. Loss in production rates and obsolescence were deemed to be insignificant by knowledgeable persons within the company.

Property B Data obtained for Property B were identical to that for Property A, except that the repair and maintenance costs were available for only three years for all units. A tabular summary and graphical plot of these data for an average unit are shown in Table 9 and Figure 12 respectively. The probable service life for Property B was estimated by company sources to be approximately 30 years.

Property C The property units contained in Property C were quite different from the first two property groups in several respects. First, the presses contained in this group were larger, more complex pieces of equipment that had a significantly higher original cost than did Properties A and B. As a result, Property C units were more prone to incur "capital improvement" costs that complicated the tabulation of repair and maintenance costs. Second, the variability among individual units was larger than among units in Properties A and B. Nonetheless, the data available for this property were similar to that available for the first two properties. A tabular summary and graphical plot of the

Table 8. Summary of repair and maintenance expenses for Property A

Age (yrs)	Annual repair and maintenance cost ($/hr)
1	1847
2	2491
3	2800
4	2698
5	3301
6	2918
7	1994
8	1967
9	1818
10	1830
11	1678
12	2102
13	1972
14	2290
15	2289
16	2447
17	2432
18	1155
19	1549

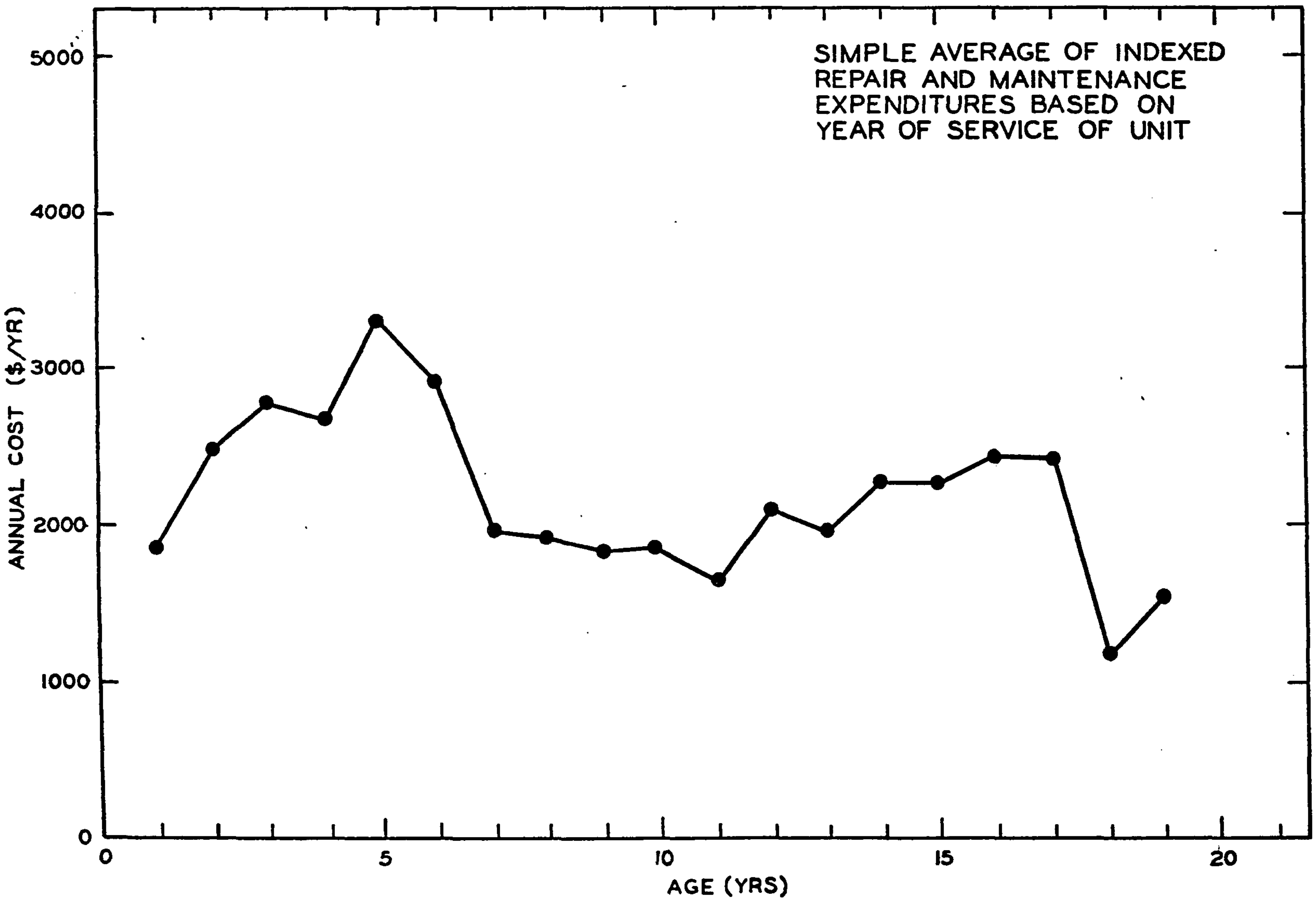

Fig. 11. Repair and maintenance costs for Property A

Table 9. Summary of repair and maintenance expenses for Property B

Age (yrs)	Annual repair and maintenance cost ($/hr)
1	1899
2	69
3	60
4	470
5	1743
6	608
7	489
8	419
9	945
10	691
11	582
12	672
13	466
14	200
15	275
16	49
17	674
18	698
19	542
20	---
21	---
22	45
23	483
24	129

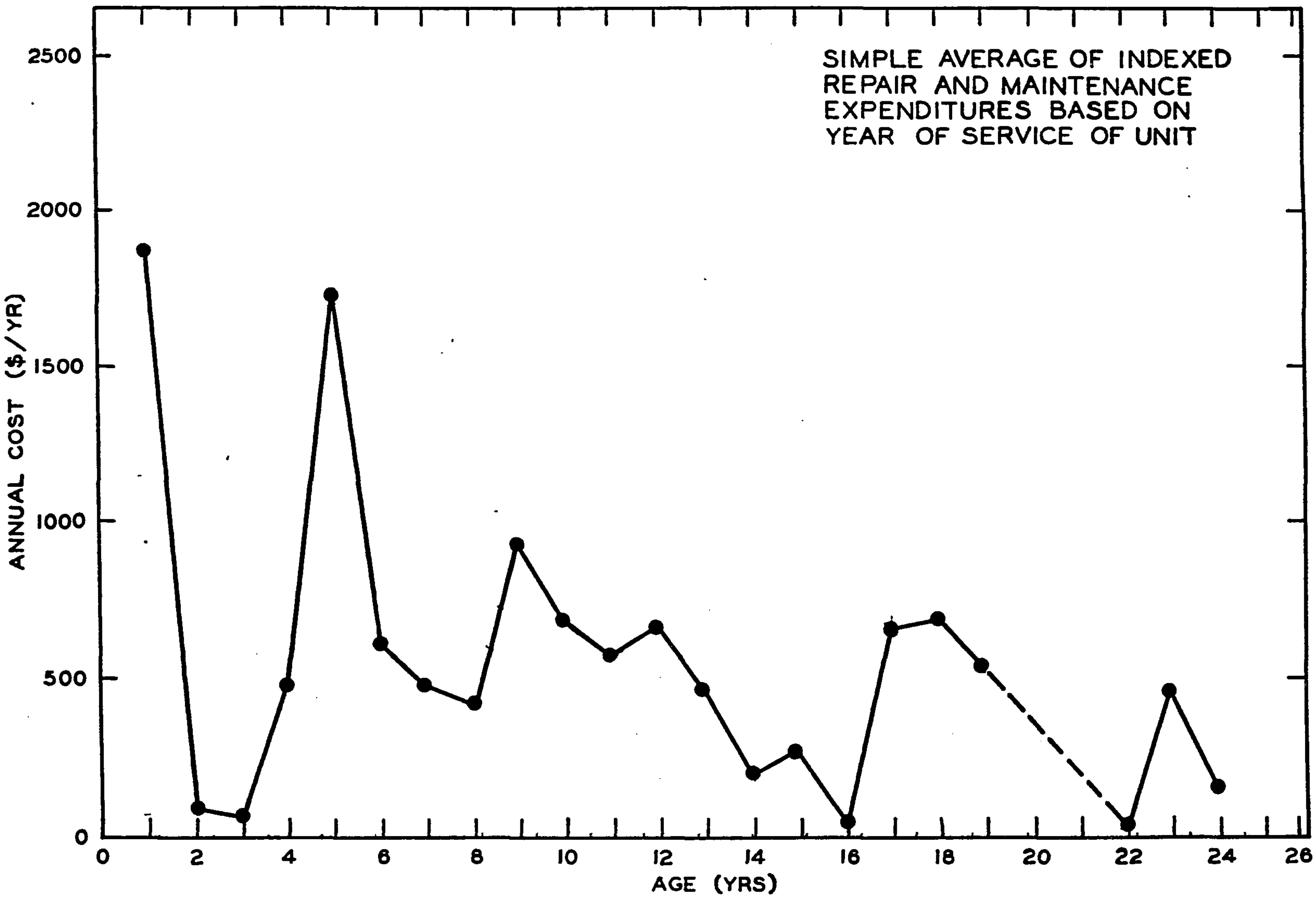

Fig. 12. Repair and maintenance costs for Property B

depreciation data for Property C are shown in Table 10 and Figure 13 respectively. The probable service life for an average unit was estimated to be 20 years.

Property D Property D was similar to Properties A, B, and C in that it was an industrial property. It was different, however, in that it was not a production unit that functioned as a part of an assembly line. Rather, it was composed of mobile units supporting one or more of the assembly lines. Nonetheless, the data available and characteristics exhibited by Property D units were similar to those of Properties A, B, and C. One notable exception to the similarity of data available was the significant, and increasing, downtime known to occur as the units aged. Since a disabled unit could be replaced with a backup unit, no tradeoff of expensive repair and maintenance costs for minimum downtime existed. Unfortunately, though it was known that downtime occurred, company records were not complete enough to determine the downtime cost. Therefore, as with most situations where adequate data were not available, the accuracy of the T-factor estimation suffered.

A tabular summary and graphical plot of the depreciation data for Property D are shown in Table 11 and Figure 14 respectively. The probable service life was estimated to be 15 years for the units composing Property D.

Property E Data collected for Property E were identical to that for Property D. A tabular summary and graphical plot of the depreciation data are shown in Table 12 and Figure 15 respectively. The probable

Table 10. Summary of repair and maintenance expenses for Property C

Age (yrs)	Annual repair and maintenance cost ($/hr)
1	24,285
2	39,226
3	39,381
4	32,264
5	37,673
6	46,262
7	52,897
8	43,944
9	88,859

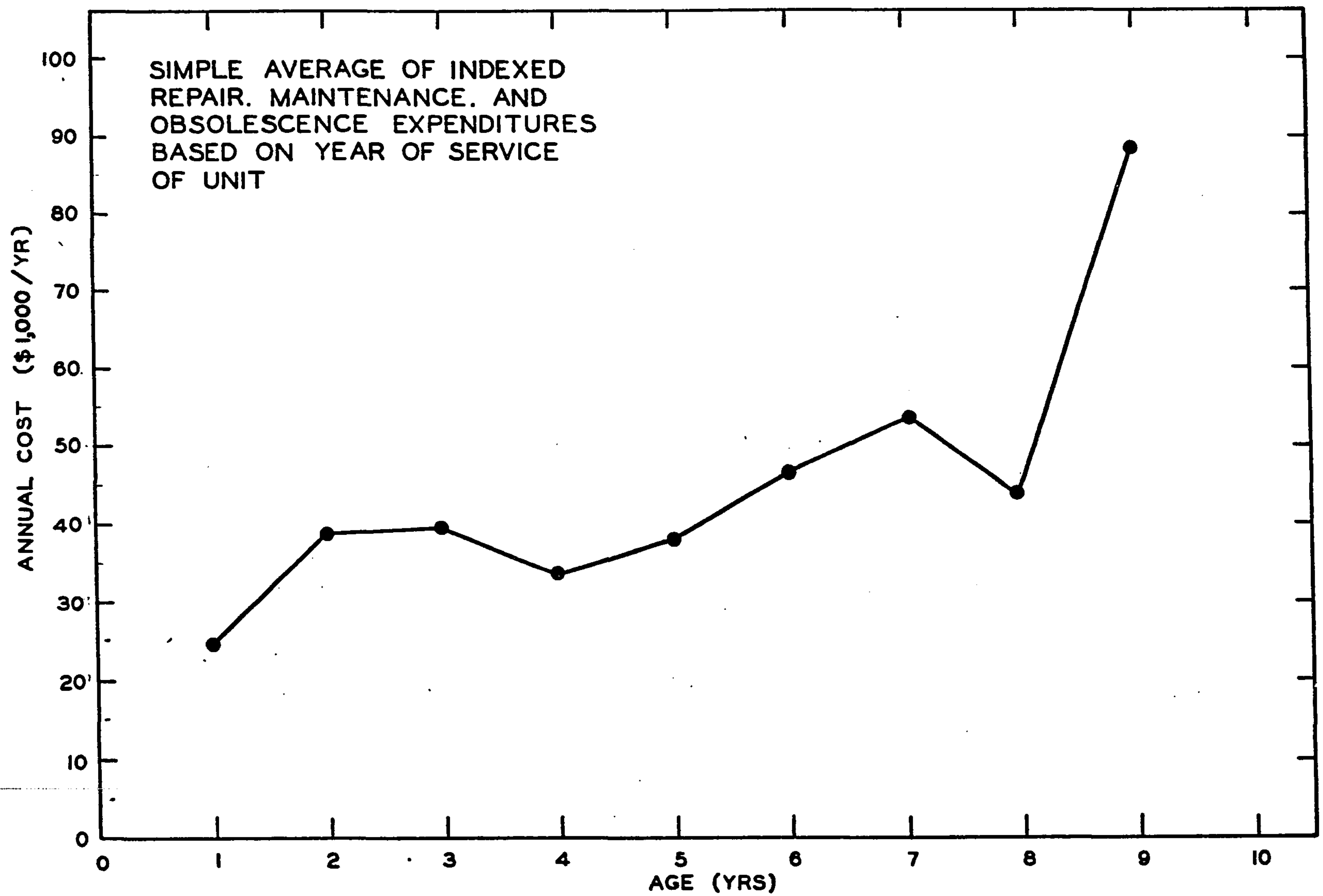

Fig. 13. Repair, maintenance, and obsolescence costs for Property C

Table 11. Summary of repair and maintenance expenses for Property D

Age (yrs)	Annual repair and maintenance cost ($/hr)
1	7,479
2	7,424
3	13,363
4	11,553
5	12,171
6	11,653
7	14,722
8	14,138
9	11,188
10	12,942
11	6,485
12	5,856
13	4,738

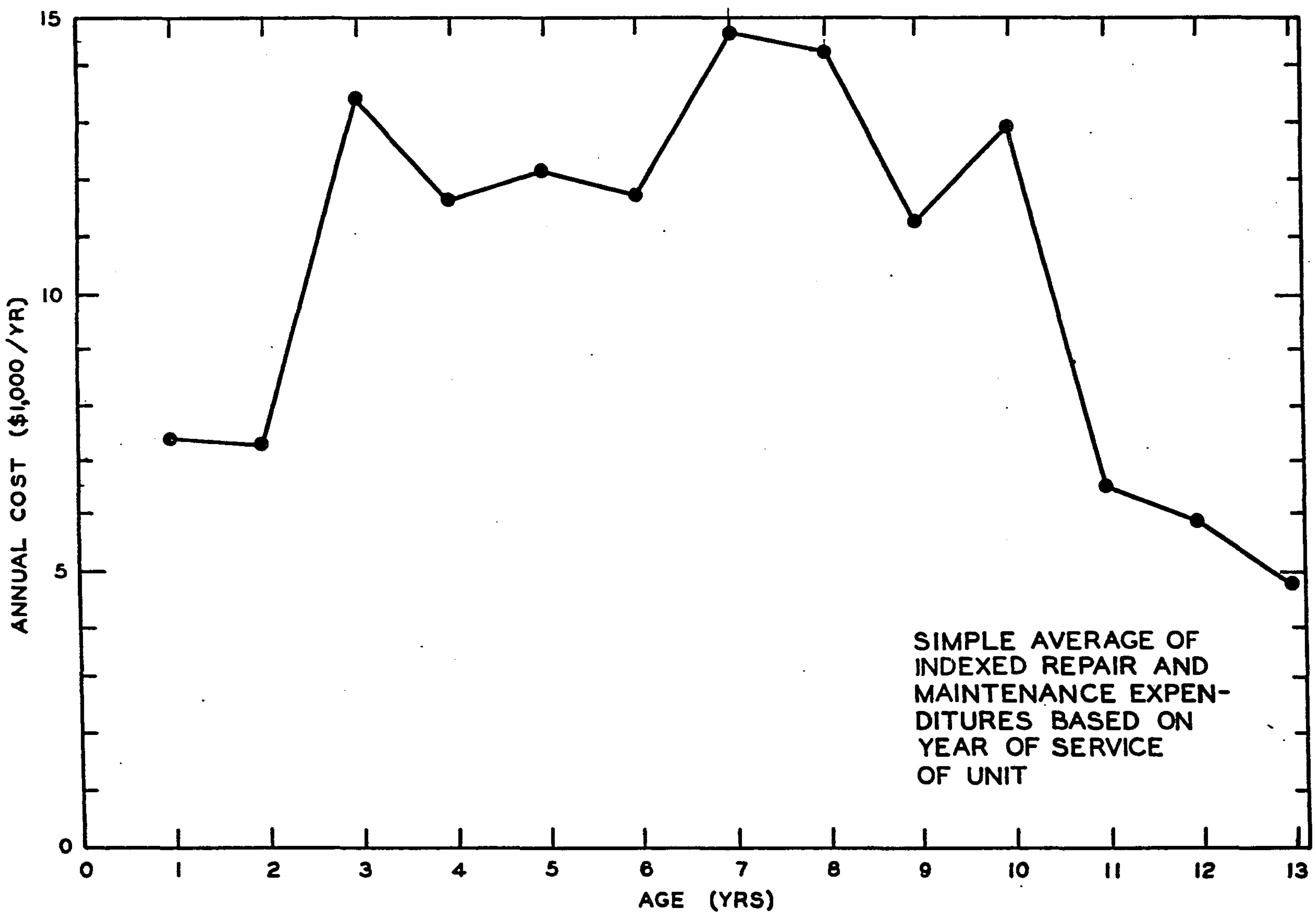

Fig. 14. Repair and maintenance costs for Property D

Table 12. Summary of repair and maintenance expenses for Property E

Age (yrs)	Annual repair and maintenance cost ($/hr)
1	33,415
2	29,799
3	48,882
4	40,340
5	46,518
6	55,670
7	41,835
8	83,195
9	62,667

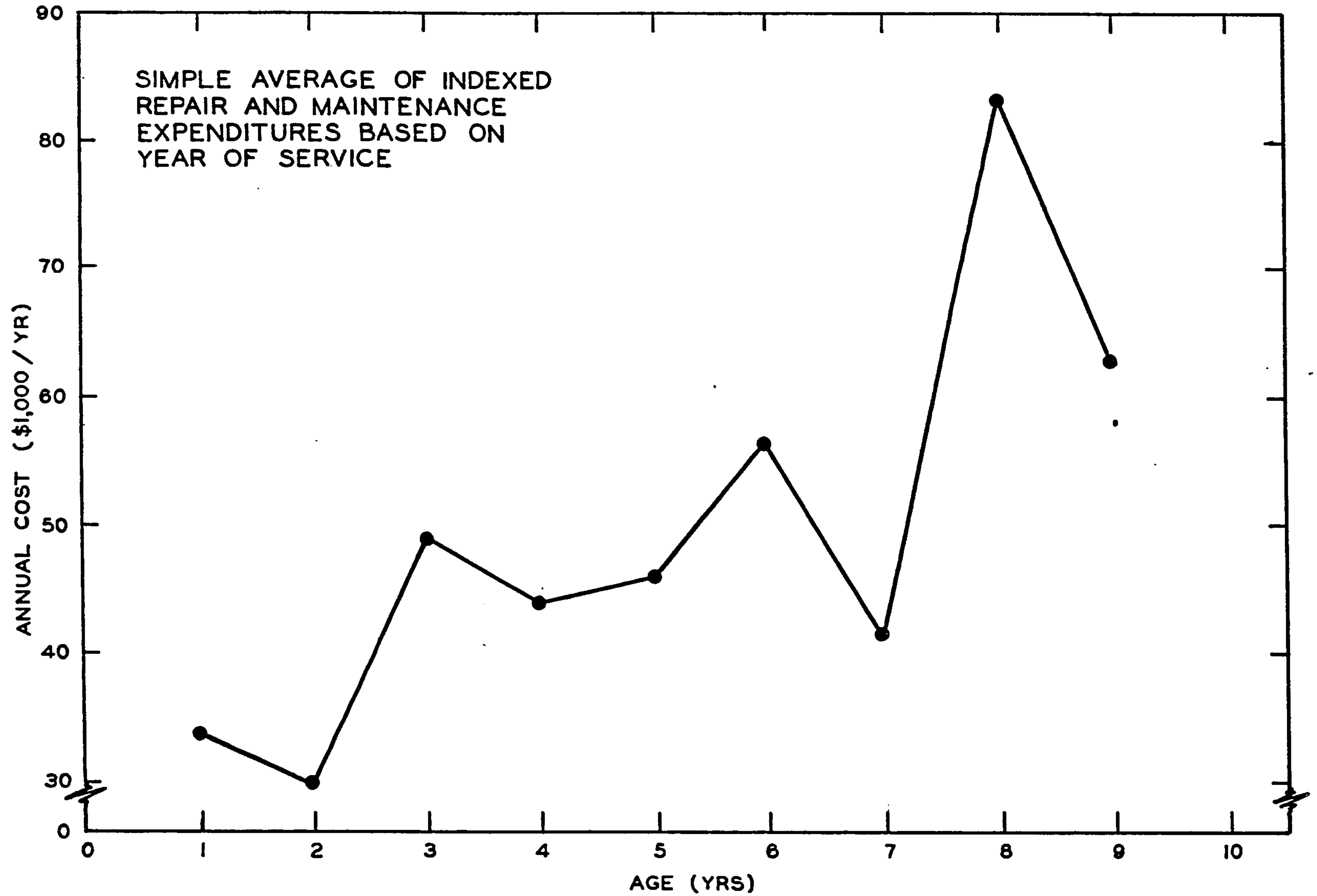

Fig. 15. Repair and maintenance costs for Property E

service life was estimated to be 10 years for the units composing Property E.

Experimental Procedure Followed

Utilizing the addition data collected for the D8 dozer, 3/4 ton pickup, and industrial forklift properties, T-factor values were estimated using the proposed Delta Procedure. The resulting T-factor values were then compared to the T-factor values estimated using the market evidence data. A favorable comparison would strongly support the validity of the Delta Procedure.

Data collected for Properties A, B, C, D, and E were used to estimate T-factor values for those properties. The estimated T-factor values were then used to compute value vs. age results for each property. Since these values could not be obtained by an alternate procedure, no means of substantiating the results was available.

ANALYSIS AND DISCUSSION OF RESULTS

Using the experimental procedures described and data collected, T-factor values for a number of properties were estimated. In addition, several tests were performed to ascertain the effects of varying some of the input parameters. The experimental results obtained were, in general, comparable to those anticipated.

T-factors for Known Properties Using Data Fitting Techniques

One of the objectives of this study was to estimate a T-factor value for a select number of properties using market evidence data. Therefore, collected data were analyzed and T-factor values estimated for three select properties: D8 dozers, 3/4 ton pickups, and industrial forklifts.

D8 Dozers

Data collected for the D8 dozer property did not include an Iowa type curve. Application of a computer based, trial and error procedure to select the best curve resulted in the L5, S6, or R5 Iowa type curves being selected as the best fitting curves. All three curves were found to fit with about the same degree of excellence.

Further analysis of the data, however, revealed that, though the degree of fit between the computed curve and the market evidence data was better for some curve types than for other, the best fit for each curve always resulted in the selection of the same value of T. In other words, the Iowa type curve selection affected the closeness of fit, but made no apparent difference in the T-factor value selected as being the best estimate.

Figure 16 shows the range of computed value curves for an array of Iowa type curve selections with a constant value of T. The relatively small amount of variance shown demonstrates the small effect that curve selection has on the final result.

The rate of return was selected to be 7% per annum for all calculations in this study (25). Since the most reasonable value for an inflation-free, annual rate of return varies, an analysis was made to determine what effect varying the discount rate would have on the computed value curve. The value curves resulting from holding all parameters in the Elfar model constant, except the rate of return, are shown in Figure 17. Based on this graphical plot, the rate of return term had little effect on the completed equipment values as long as a reasonably accurate rate was used. This lack of sensitivity supported the acceptability of using a constant annual rate of $r = 7\%$ for all calculations.

Based on the data collected, and the experimental results described above, the T-factor value for D8 dozers was estimated by fitting a computed value curve to the market evidence data. The best fit was found for a value of $T = .91$.

<u>3/4 Ton Pickups</u>

Matching the stub curve shown in Figure 6 against a standardized set of Iowa type curves identified the pickup property as probably exhibiting an L2-17, S1.5-17, or R3-13 survivor curve. Though the visual matches were about equal for all three curve types, the R3-13 curve was selected

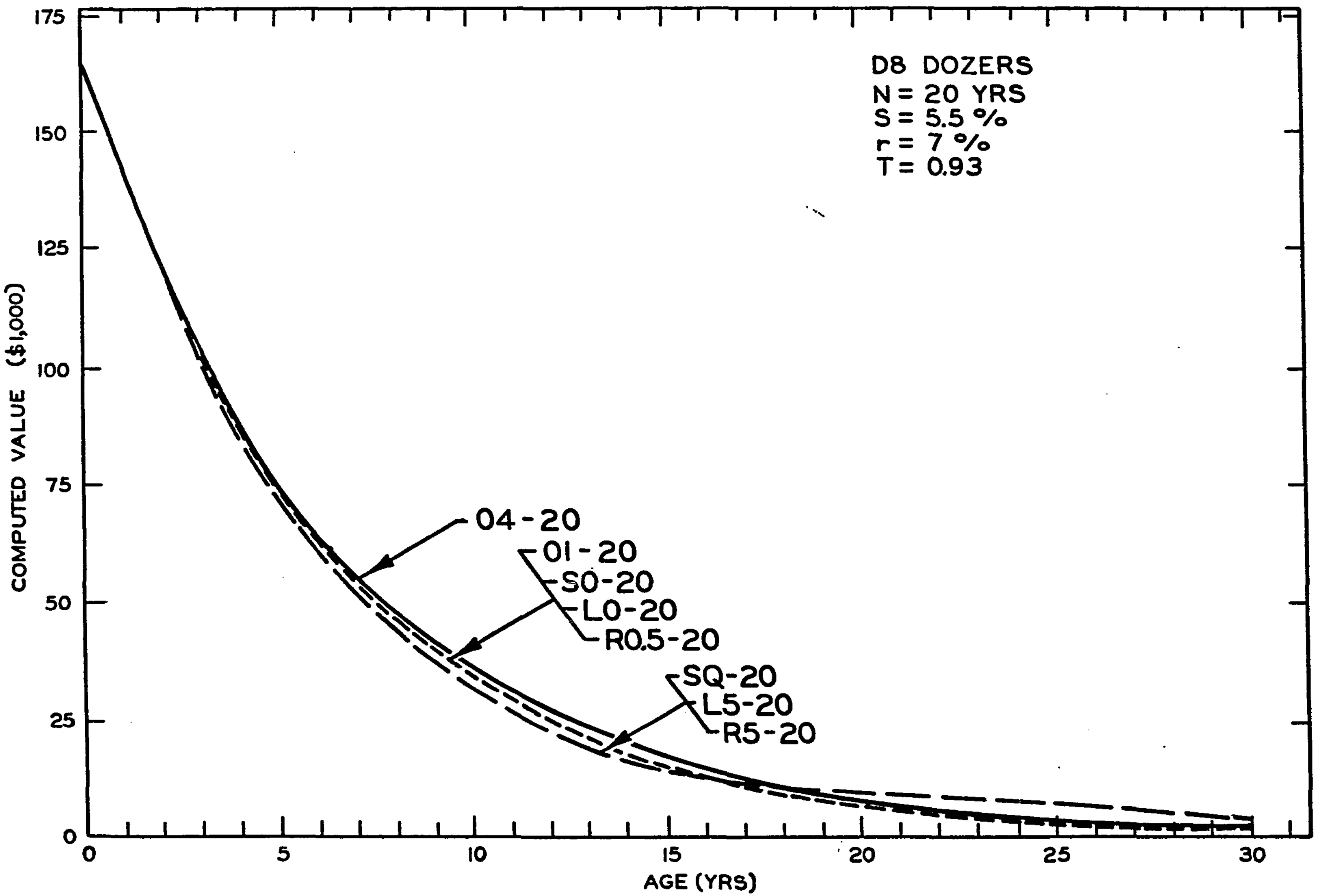

Fig. 16. Effects of Iowa type curve selection on computed value curves

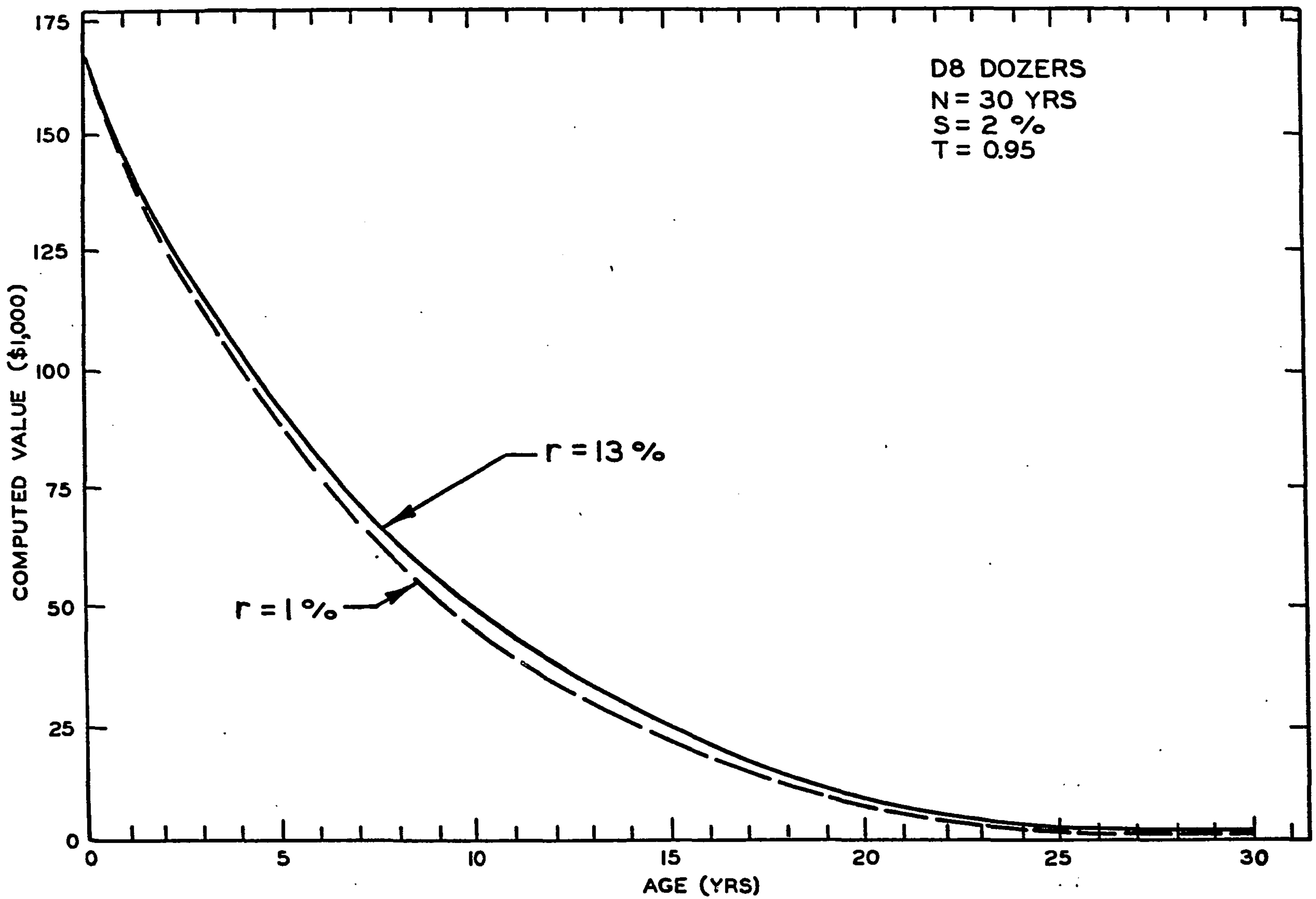

Fig. 17. Effect of varying rate of return on computed value curves

as being the most reasonable of the group. As with the D8 dozer property, however, computer analysis demonstrated that, regardless of the curve type selected, the best T-factor value was the same.

Analysis of the data collected, in addition to the results presented above, resulted in two estimates for the T-factor. Based on the NADA data source, the best estimate was T = .95. Based on the construction contractor source, however, the best estimate was T = .80. Since neither value was known to be superior at this point, both were retained for later use.

Industrial Forklifts

Selection of the most appropriate Iowa type curve, as for the D8 dozer property, was reduced to a trial and error curve fitting procedure. Based on this procedure, the L5, S6, and R5 curves were selected as giving the best fit of the computed value curve to the market evidence data. Further analysis concluded, as it had with the other two properties, that the Iowa type curve selected had little effect on the T-factor value estimation. Only the closeness of fit was affected by the Iowa type curve used.

The result of the trial and error curve fitting procedure applied to the data collected and analyzed for forklift account was found to be a range of values for the progression rate. This range was between T = 1.02 and T = 1.04.

T-factors for Known Properties Using the Delta Procedure

The proposed Delta Procedure was applied to the D8 dozer, pickup, and forklift properties to estimate progression rate values. The T-factor values estimated using the Delta Procedure were found to approximate, with reasonable accuracy, the T-factor values found earlier.

The first step of the proposed Delta Procedure was to quantify P (periodic reduction in operation returns) as completely as possible. Using data collected from the construction contractor source (22), the histogram shown in Figure 18 was produced for the D8 dozer property. Division of the histogram into strips of semiannual width allowed the computation of a P term for each period. The width of each strip, 700 machine hours, was the national average for D8 dozer usage (30, p. 38). The P term for the first period, as an example, was computed as (700 machine-hours/period) X ($15.39/machine-hour) = $10,773/period. A tabular presentation of the P terms for D8 dozers is contained in Table 13.

Using a procedure similar to that used for the D8 dozer property, P was also quantified for the 3/4 ton pickup property. Data obtained from the construction contractor source (22) were compiled to produce the histogram shown in Figure 19. Dividing this histogram into 900 machine-hour, semi-annual strips (22), and computing the areas of each strip, produced the values for the P terms shown in Table 14.

The industrial forklift property data for repair and maintenance and downtime are presented in Table 7. Using these data, the histogram in Figure 20 was produced. The computed P terms for the forklift property are contained in Table 15.

The second step of the proposed Delta Procedure was to compute Δ.

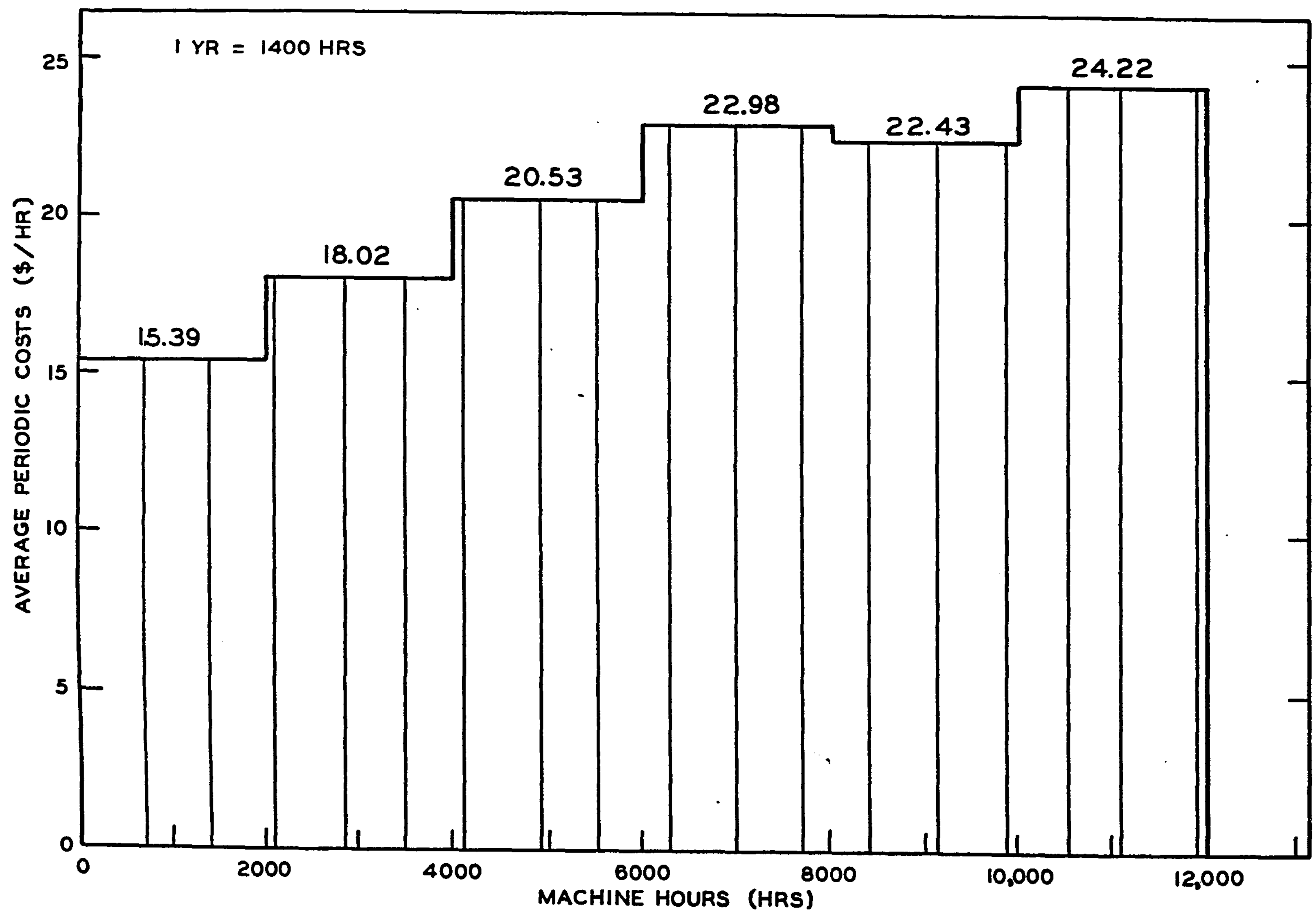

Fig. 18. Average periodic costs vs. machine hours for D8 dozers

Table 13. Computation of Delta Procedure parameters for D8 dozers

Age (yrs)	Cumulative machine hours	P ($)	Delta (Δ)	Delta ratio
0.5	700	10,773	0	.000
1.0	1400	10,773	0	.000
1.5	2100	11,036	263	.002
2.0	2800	12,614	1841	.011
2.5	3500	12,614	1841	.011
3.0	4200	13,116	2343	.014
3.5	4900	14,371	3598	.022
4.0	5600	14,371	3598	.022
4.5	6300	15,106	4333	.026
5.0	7000	16,086	5313	.032
5.5	7700	16,086	5313	.032
6.0	8400	15,866	5093	.031
6.5	9100	15,701	4928	.030
7.0	9800	15,701	4928	.030
7.5	10500	16,596	5823	.035
8.0	11200	16,954	6181	.038
8.5	11900	16,954	6181	.038

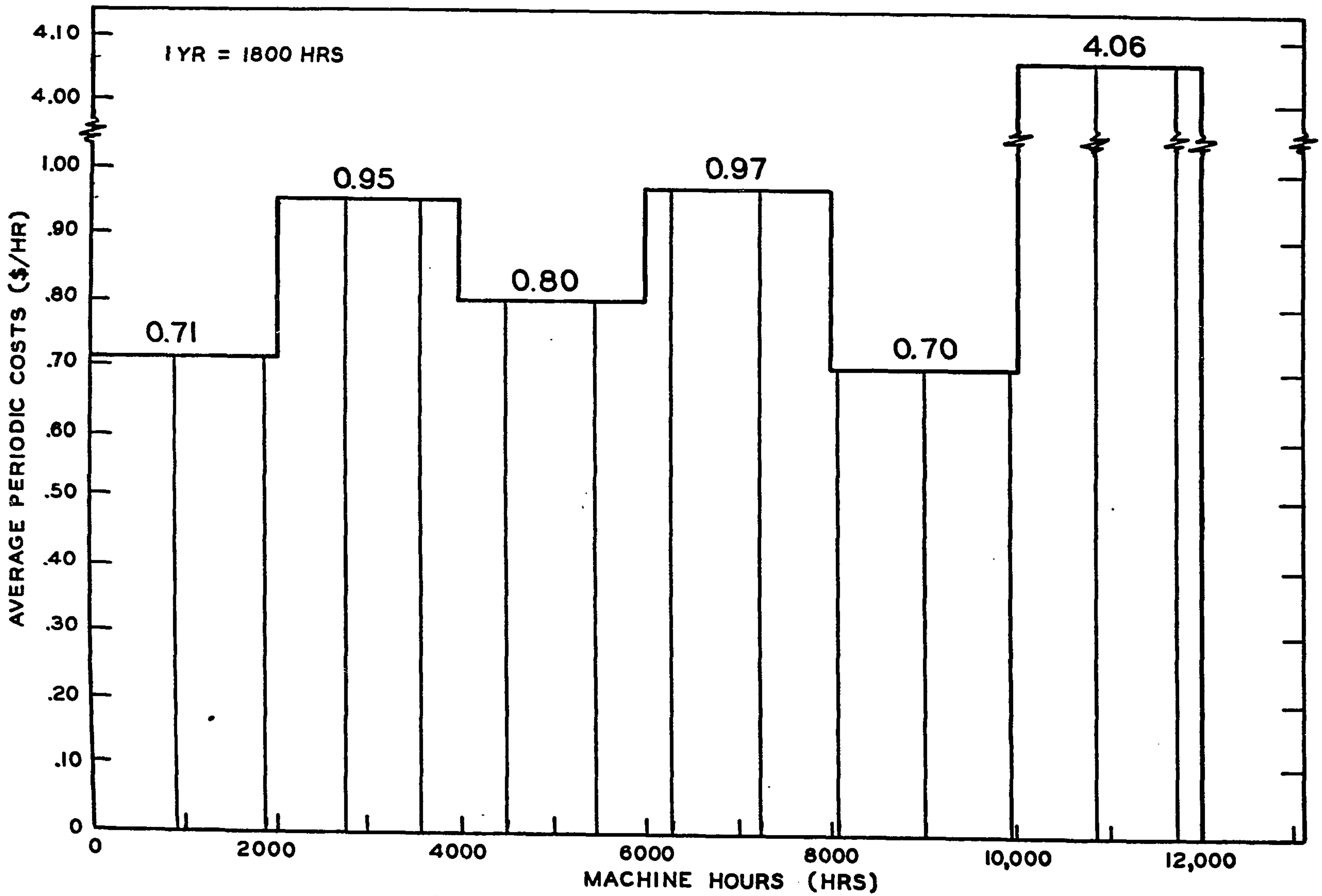

Fig. 19. Average periodic costs vs. machine hours for pickup trucks

Table 14. Computation of Delta Procedure parameters for pickup trucks

Age (yrs)	Cumulative machine hours	P ($)	Delta (Δ)	Delta ratio (NADA)	Delta ratio (contractor)
0.5	900	639	0	.000	.000
1.0	1800	637	0	.000	.000
1.5	2700	807	168	.031	.026
2.0	3600	807	168	.031	.026
2.5	4500	780	141	.026	.022
3.0	5400	720	81	.015	.013
3.5	6300	771	132	.024	.020
4.0	7200	873	234	.043	.036
4.5	8100	846	207	.038	.032
5.0	9000	630	- 9	----	----
5.5	9900	630	- 9	----	----
6.0	10800	3318	2679	.492	.414
6.5	11700	3654	2679	.553	.466

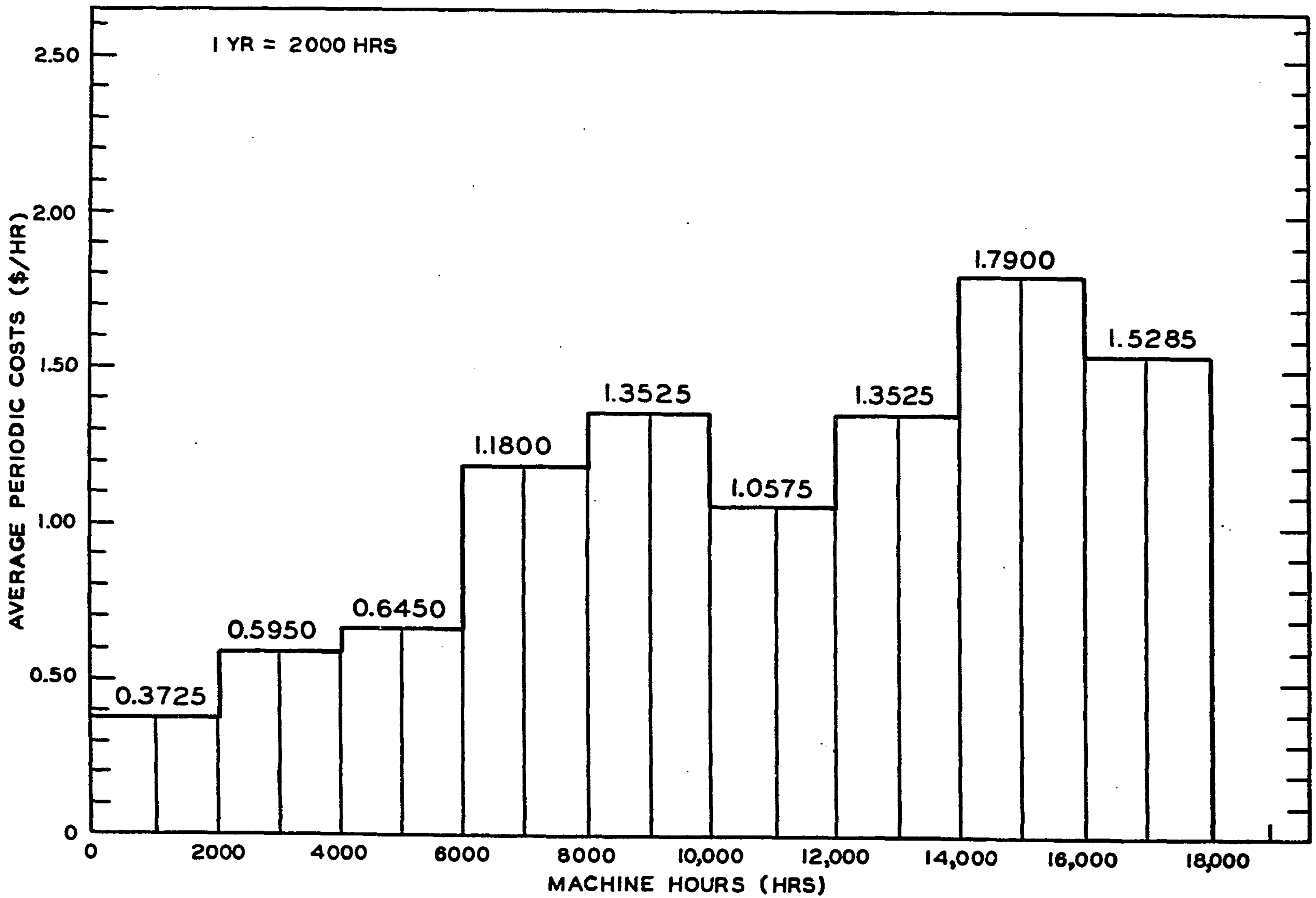

Fig. 20. Average periodic costs vs. machine hours for industrial forklifts

Table 15. Computation of Delta Procedure parameters for forklifts

Age (yrs)	Cumulative machine hours	P ($)	Delta (Δ)	Delta ratio
0.5	1,000	372	0	.000
1.0	2,000	373	1	.000
1.5	3,000	595	223	.012
2.0	4,000	595	223	.012
2.5	5,000	645	273	.015
3.0	6,000	645	273	.015
3.5	7,000	1180	808	.044
4.0	8,000	1180	808	.044
4.5	9,000	1352	980	.053
5.0	10,000	1353	981	.054
5.5	11,000	1057	685	.037
6.0	12,000	1058	686	.037
6.5	13,000	1352	980	.053
7.0	14,000	1353	981	.054
7.5	15,000	1790	1418	.077
8.0	16,000	1790	1418	.077
8.5	17,000	1528	1156	.063
9.0	18,000	1529	1157	.063

Computations of this term are contained in Tables 13, 14, and 15 for the D8 dozer, pickup, and forklift properties respectively.

The third step of the proposed Delta Procedure was to determine V_N. Based on original cost information, and as previously presented, the value new of the D8 dozers was determined to be $165,000.00 (23), the 3/4 ton pickup trucks $6474.00 (22) or $5450.00 (26), and the forklifts $18,350.00 (29).

The fourth step of the proposed procedure was to compute the Delta Ratio, (Δ/V_N). Completion of the second and third steps above reduces this to a matter of simple mathematics. The results for the D8 dozers, pickups, and forklifts are again given in Tables 13, 14, and 15 respectively.

The fifth step of the proposed Delta Procedure was to determine the probable service life. These quantities have been previously determined to be 20 years (D8 dozers), 13 years (pickups), and 10 years (forklifts).

The sixth, and final, step of the proposed Delta Procedure was to estimate the value of T using a standard set of curves. Three sets of curves based on the appropriate parameter values for the three properties are shown in Figures 21, 22, and 23. Also shown in these figures are the observed Delta Ratio values computed in Tables 13, 14, and 15. Based on comparison of the observed points with the standard curves, approximate T-factor values for the three property groups were estimated. The T-factor values estimated using the proposed Delta Procedure, and those estimated using the market evidence data, are summarized in Table 16.

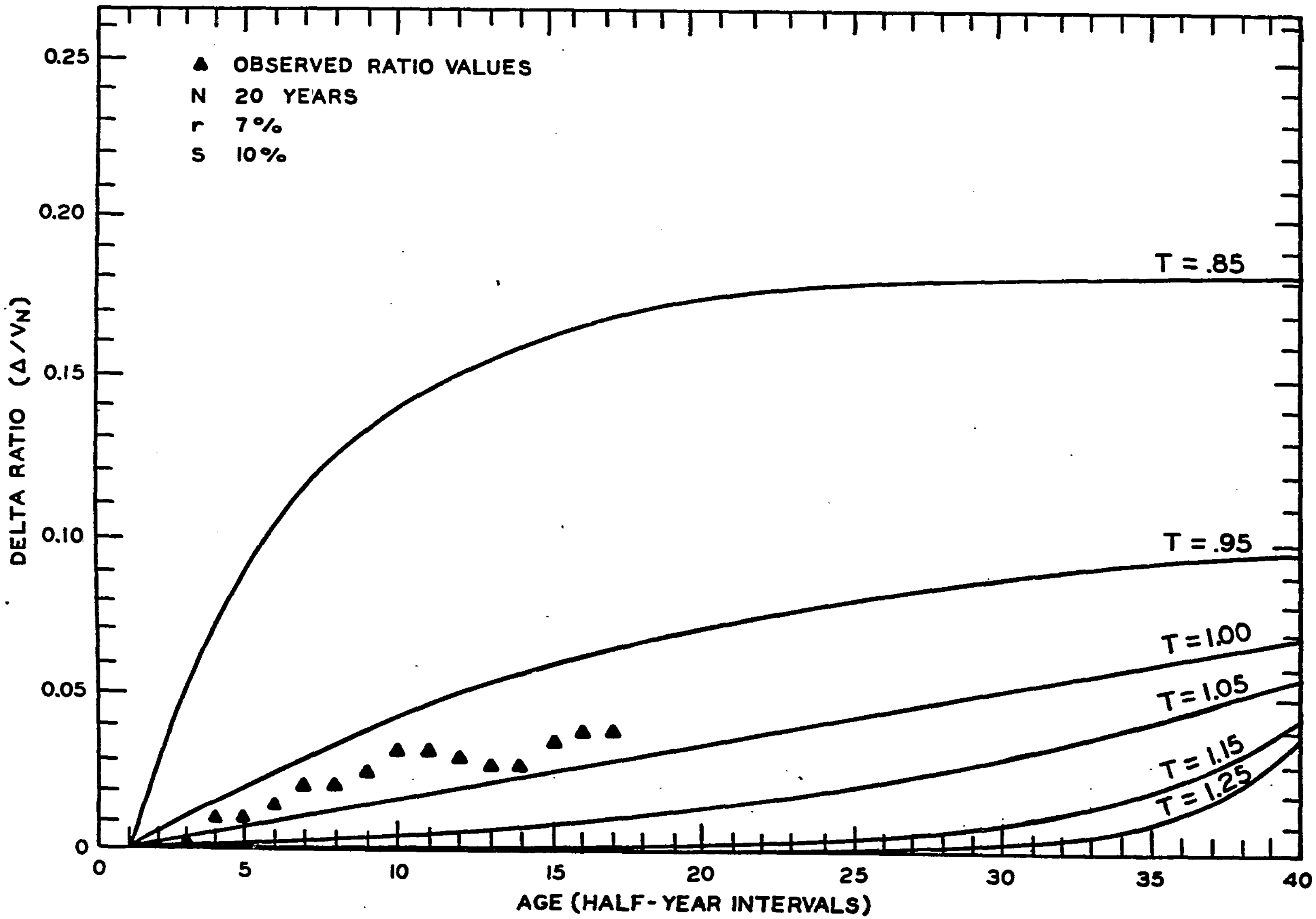

Fig. 21. Standard curves for D8 dozers

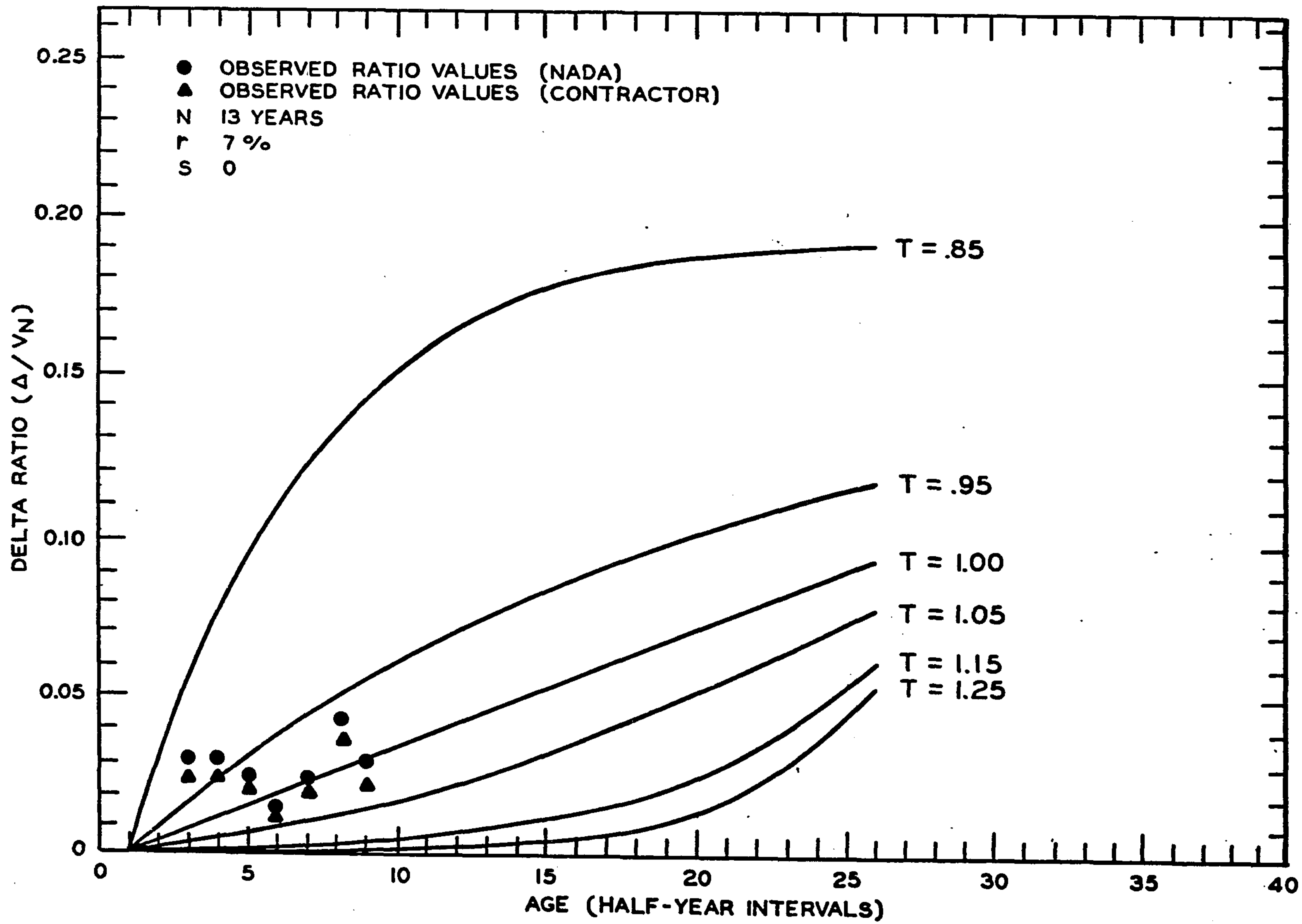

Fig. 22. Standard curves for pickup trucks

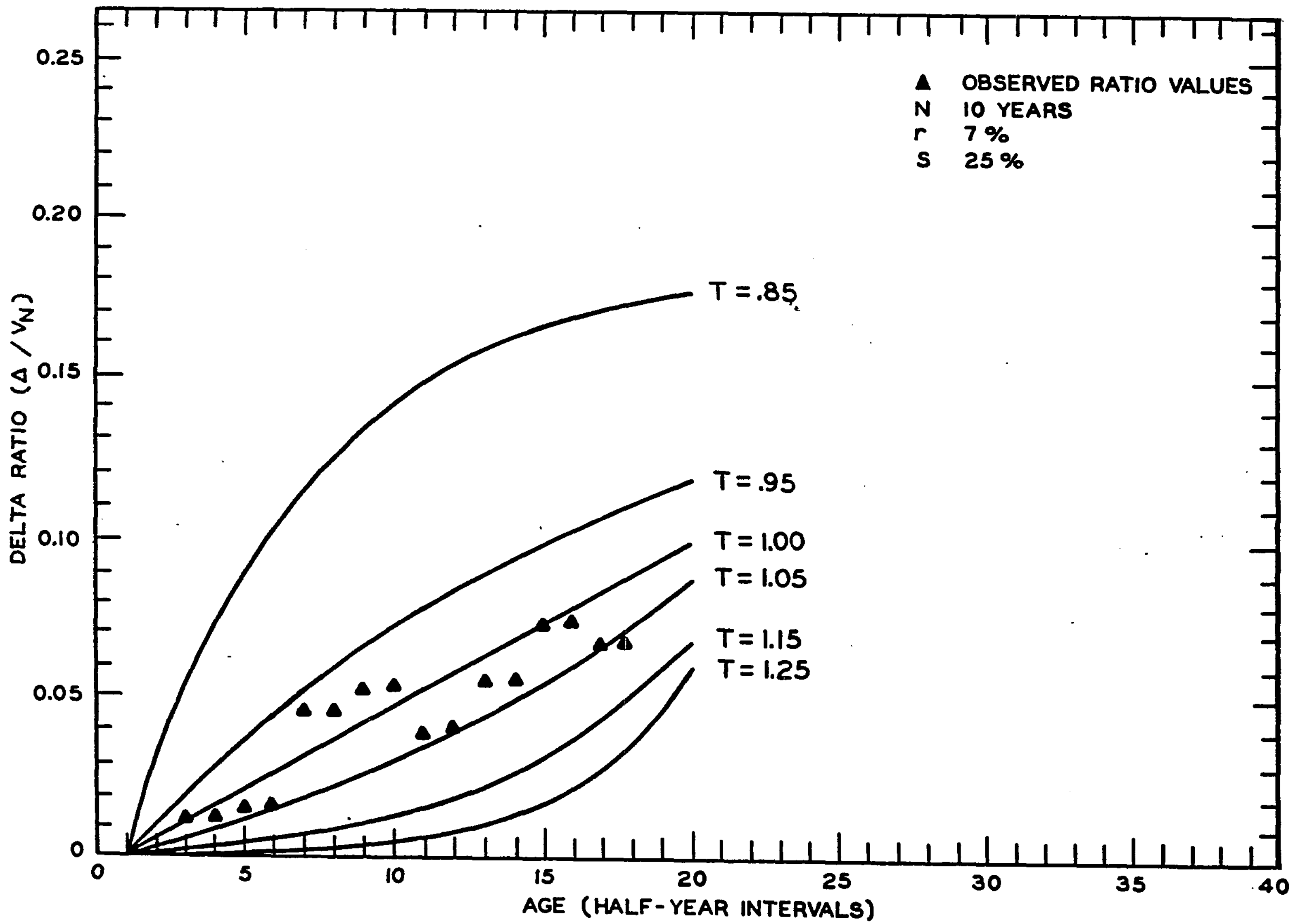

Fig. 23. Standard curves for industrial forklifts

Table 16. Summary of T-factor values for dozers, pickups, and forklifts estimated using the proposed Delta Procedure and the market evidence data

Property	Delta Procedure T-factor values	Market evidence T-factor values
D8 dozers	.96 - .98	.91
pickups	.91 - 1.00	.80[a] .95[b]
forklifts	1.00 - 1.04	1.02 - 1.04

[a] Based on contractor market evidence data
[b] Based on NADA market evidence data

Comparison of the values obtained using both procedures revealed that the proposed Delta Procedure estimated virtually the same T-factor values as those found using the market evidence data. The greatest deviation was for the pickup truck property when the T-factor estimation was based on the construction contractor source of market evidence data. The large deviation, however, only served as a reminder that even when applying the outwardly rigorous proposed procedure, judgement and knowledge of the property being valued were still required. Examination of Figure 5 revealed that the market evidence curve fell off much quicker for the contractor data source than for the NADA data source. This accelerated loss in value was determined to be a direct result of the harsher use that construction pickups receive as compared to the average pickup reflected in the NADA market evidence data. Since the repair and maintenance data shown in Figure 9 indicated a substantial amount of deferred work, the quantified P term did not adequately reflect the degree of depreciation over time. In effect, the hard working, much abused construction pickup accepted a larger than normal loss in value in return for an average level of repair and maintenance expenditures. Therefore, an experienced valuation engineer who is familiar with the property being valued, may have to judgementally reduce the T-factor if he finds that the P term does not reflect the true loss in operation returns.

T-factors for Iowa Properties Using the Delta Procedure

The original problem that initiated this study was concern over possible inadequacies in the procedures being used to value industrial equipment owned and operated by several Iowa firms. Though the Elfar model was available, and appeared sound in its derivation, no means of actually applying the model had been developed. Based on the discussion and derivations presented, however, a possible procedure has now been proposed and tested. Application of the proposed Delta Procedure to the data collected for Properties A, B, C, D, and E was therefore the final phase of this study.

Property A

Estimation of the T-factor value for Property A was accomplished using the following procedure.

Quantify P   The P term quantities contained in Table 17 were derived from the information shown in Table 8.

Compute Δ Results of the Δ term calculations are shown in Table 17.

Determine V_N The value new for Property A was set equal to the average of the trended original costs of the 16 individual units. This value was found to be $49,963.

Compute Δ/V_N The results of this computation are shown in Table 17.

Determine the Probable Service Life   The probable service life was previously estimated to be approximately 30 years.

Estimate the value of T   Estimation of the T-factor value for Property A utilized the standard set of curves plotted in Figure 24 and the Δ/V_N values contained in Table 17. Examination of Figure 24 produced an estimated T-factor value in the range of .95 to 1.00 for the first five years of life, then a rise to 1.06 to 1.08 beyond. The lower T-factor values were caused by higher than normal P quantities. The higher P quantities appear to have resulted from shakedown costs incurred as each unit was put into service. As the unit of equipment aged, a higher value of the T-factor soon predominated. Based on this knowledge of equipment characteristics, and applying a bit of judgement, the T-factor value for Property A was estimated to be between T = 1.06 and T = 1.08.

Property B

Estimation of the progression rate for Property B using the proposed Delta Procedure produced the following results.

Table 17. Computation of Delta Procedure parameters for Property A

Age (yrs)	P ($)	Delta (Δ)	Delta ratio
0.5	923	0	.000
1.0	924	1	.000
1.5	1245	322	.006
2.0	1246	323	.006
2.5	1400	477	.010
3.0	1400	477	.010
3.5	1349	426	.009
4.0	1349	426	.009
4.5	1650	727	.015
5.0	1651	728	.015
5.5	1459	536	.011
6.0	1459	536	.011
6.5	997	74	.001
7.0	997	74	.001
7.5	983	60	.001
8.0	984	61	.001
8.5	909	- 14	----
9.0	909	- 14	----
9.5	915	- 8	----
10.0	915	- 8	----
10.5	839	- 84	----
11.0	839	- 84	----
11.5	1051	128	.003
12.0	1051	128	.003
12.5	986	63	.001
13.0	986	63	.001
13.5	1145	222	.004
14.0	1145	222	.004
14.5	1144	221	.004
15.0	1145	222	.004
15.5	1223	300	.006
16.0	1224	301	.006
16.5	1216	293	.006
17.0	1216	293	.006
17.5	577	-346	----
18.0	578	-345	----
18.5	774	-149	----
19.0	775	-148	----

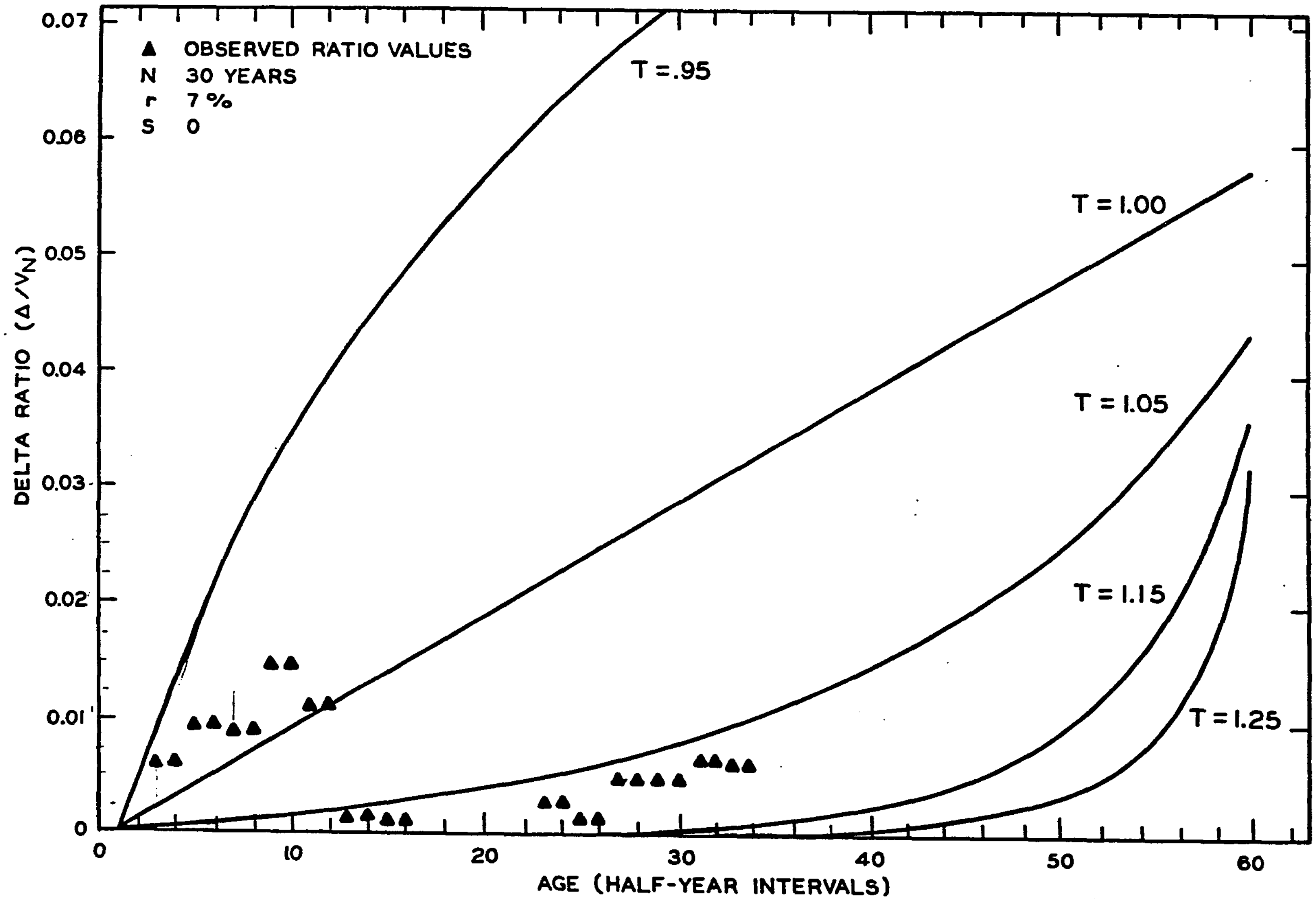

Fig. 24. Standard curves for Property A

Quantify P The quantification of the P term was based on data presented in Table 9 and Figure 12. Resulting quantities are contained in Table 18. Since the available data covered only a three year band for each property unit, the average annual repair and maintenance cost was often based on just one of the 17 units. The result was a cost that was not as reliable as one having a contribution from all 17 units.

Another irregularity of the Property B data was that the highest annual repair and maintenance cost occurred during the first year. If the Δ values were computed based on this point, then all Δ values would be negative. Since T-factor values are only defined for $\Delta > 0$, negative values of Δ lead to an undefined and unworkable situation. Examination of the trim press data revealed that the high, first year cost was due solely to a single unit. Closer inspection revealed that the $1899 cost was apparently caused by excessive start-up costs for that single unit. The excessive costs included $394 to repair oil leaks, $289 for electrical repairs, $604 for miscellaneous and broken parts, $545 for revisions, and $48 for miscellaneous hydraulic repairs. Review of the costs incurred during the second and third years of this unit indicated that $8, $17, $28, $12, and $8 respectively were more typical. Therefore, the first year repair and maintenance costs for the newest unit were judgementally revised downward to $92.

An alternate method for handling excessive start-up or shakedown costs during the first year is to base the Δ calculation on the P quantity observed for the second year. This alternate approach was tested for Property B and found to result in an estimated T-factor value

Table 18. Computation of Delta Procedure parameters for Property B

Age (yrs)	P ($)	Delta (Δ)	Delta ratio
0.5	46	0	.000
1.0	46	0	.000
1.5	34	- 12	----
2.0	35	- 11	----
2.5	30	- 16	----
3.0	30	- 16	----
3.5	235	189	.010
4.0	235	189	.010
4.5	871	825	.044
5.0	872	826	.044
5.5	304	258	.014
6.0	304	258	.014
6.5	244	198	.010
7.0	245	199	.011
7.5	209	163	.009
8.0	210	164	.009
8.5	472	426	.023
9.0	473	427	.023
9.5	345	299	.016
10.0	346	300	.016
10.5	291	245	.013
11.0	291	245	.013
11.5	336	290	.015
12.0	336	290	.015
12.5	233	187	.010
13.0	233	187	.010
13.5	100	54	.003
14.0	100	54	.003
14.5	137	91	.005
15.0	138	92	.005
15.5	24	- 22	----
16.0	25	- 21	----
16.5	337	291	.002

Table 18. (continued)

Age (yrs)	P ($)	Delta (Δ)	Delta ratio
17.0	337	291	.002
17.5	349	303	.002
18.0	349	303	.002
18.5	271	225	.001
19.0	271	225	.001
19.5	---	---	----
20.0	---	---	----
20.5	---	---	----
21.0	---	---	----
21.5	22	- 24	----
22.0	23	- 23	----
22.5	241	195	.010
23.0	242	196	.010
23.5	64	18	.001
24.0	65	19	.001

range identical to the range found by using the adjusted first year P quantity.

Compute Δ The resulting Δ terms based on an adjusted first period P term are contained in Table 18.

Determine V_N The best evidence of value new was found by averaging the trended original costs of the units. The result was V_N = $18,884. Since the available index used to trend the original costs only extended back to 1959, the average cost was based on the 16 newest units of Property B.

Compute Δ/V_N The results of this computation are shown in Table 18.

Determine the Probable Service Life The probable service life was previously estimated to be approximately 30 years.

Estimate the Value of T Estimation of the T-factor value was done using the standard curve set shown in Figure 25. Over the first third of the property's probable service life, the T-factor was estimated to be 1.00. Beyond 10 years of age, the T-factor rose dramatically to a range of 1.10 to 1.15. Whether this shift was due to a change in repair and maintenance policy, a general insufficiency of data, or some other factor, was unknown. Based on the lack of observed data, the characteristics of the property, and the inability to include any components of depreciation other than repair and maintenance costs, a T-factor value in the range of 1.00 to 1.05 appeared to be appropriate for this property.

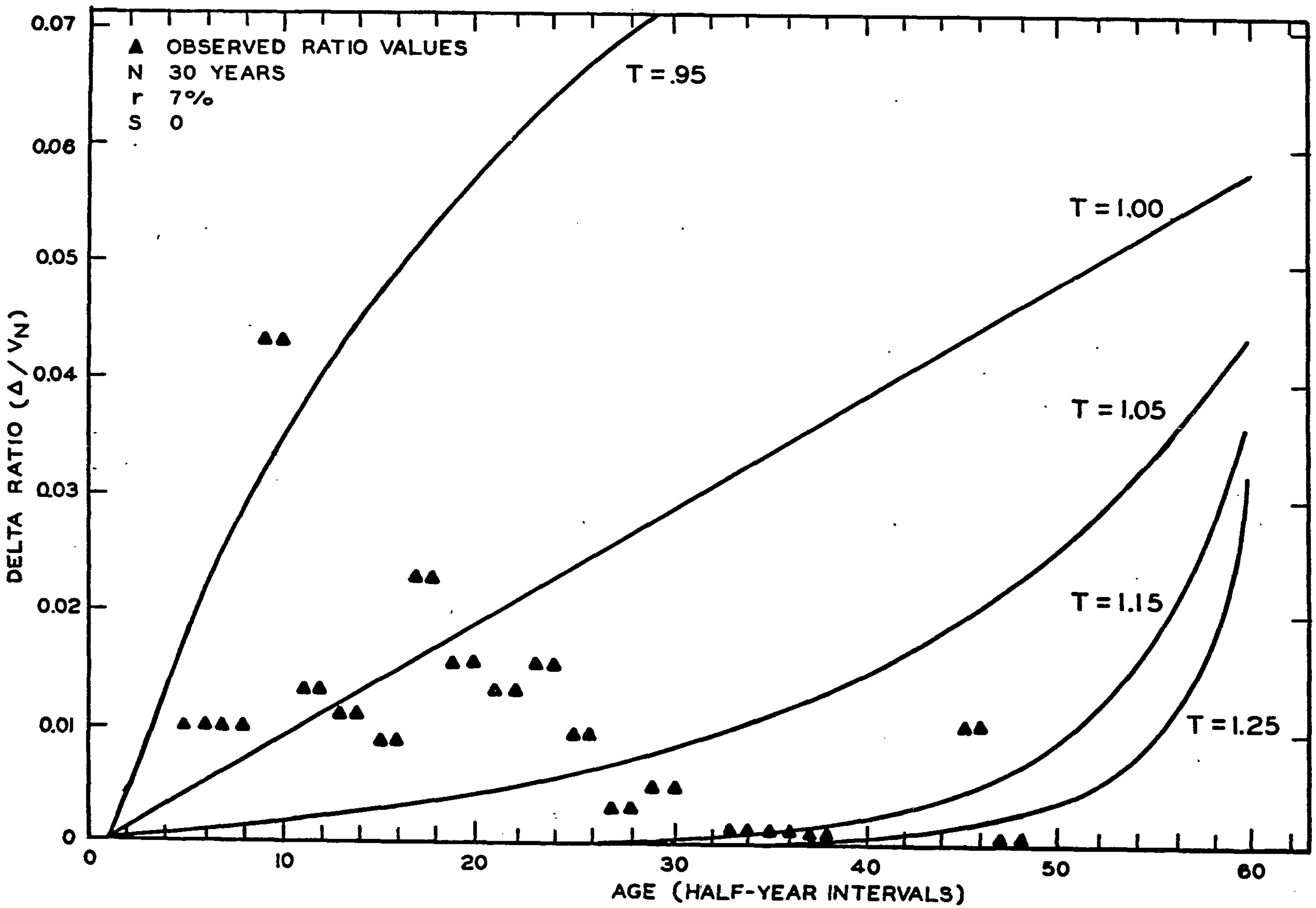

Fig. 25. Standard curves for Property B

Property C

Property C presented the largest challenge of any property handled thus far. Not only was the property composed of large, complex, and expensive units, but many of the repairs were so costly that they were capitalized rather than expensed in the accounting records. Nonetheless, Property C was analyzed using the proposed procedure, and an estimated T-factor value was found.

Quantify P A summary of the average annual P term quantities was given in Table 10 and Figure 13. The data supplied by the industrial firm not only included repair and maintenance costs, but a category of expenditure referred to as "technological costs" as well. Research into the nature of these costs revealed that the technological costs were, in effect, a measure of obsolescence. Therefore, some measure of obsolescence was combined with the repair and maintenance cost in the P term quantification. The resultant quantities for the P term are contained in Table 19.

Compute Δ The computed Δ terms are contained in Table 19.

Determine V_N Based on an average of the original costs of the 10 units, the value new was determined to be $1,126,703.

Compute Δ/V_N The computation of the Δ/V_N ratios is shown in Table 19.

Determine Probable Service Life As was previously noted, a probable service life of 20 years was estimated for Property C.

Table 19. Computation of Delta Procedure parameters for Property C

Age (yrs)	P ($)	Delta (Δ)	Delta ratio
0.5	12,142	0	.000
1.0	12,143	1	.000
1.5	19,613	7471	.007
2.0	19,613	7471	.007
2.5	19,690	7548	.007
3.0	19,691	7548	.007
3.5	16,132	3990	.004
4.0	16,132	3990	.004
4.5	18,836	6694	.006
5.0	18,837	6694	.006
5.5	23,131	10,989	.010
6.0	23,131	10,989	.010
6.5	26,448	14,306	.013
7.0	26,449	14,306	.013
7.5	21,972	9830	.009
8.0	21,972	9830	.009
8.5	44,429	32,287	.029
9.0	44,430	32,287	.029

Estimate the Value of T   Using the set of standard curves plotted in Figure 26, the T-factor was estimated to be in the range of T = 1.00 to T = 1.05.

Property D

The analysis of the data collected for Property D produced the following experimental results.

Quantify P   A summary of the average annual repair and maintenance amounts was presented in Table 11 and Figure 14. The values given in Table 11 and Figure 14 were based on extrapolated values where less than a full year's data was provided. They were also adjusted to constant dollar units using an index supplied by the data source. The values of the P terms are given in Table 20 for Property D.

Compute Δ Computation of the Δ terms are also given in Table 20 for Property D.

Determine V_N The value new for Property D, based on the average of 26 trended original costs, was determined to be $46,174.

Compute Δ/V_N Results of the Δ/V_N ratio calculations are contained in Table 20.

Determine the Probable Service Life   As was previously noted, the probable service life was estimated to be 15 years.

Estimate the Value of T   Using the quantities contained in Table 20, and the standard curve set shown in Figure 27, a T-factor value of about

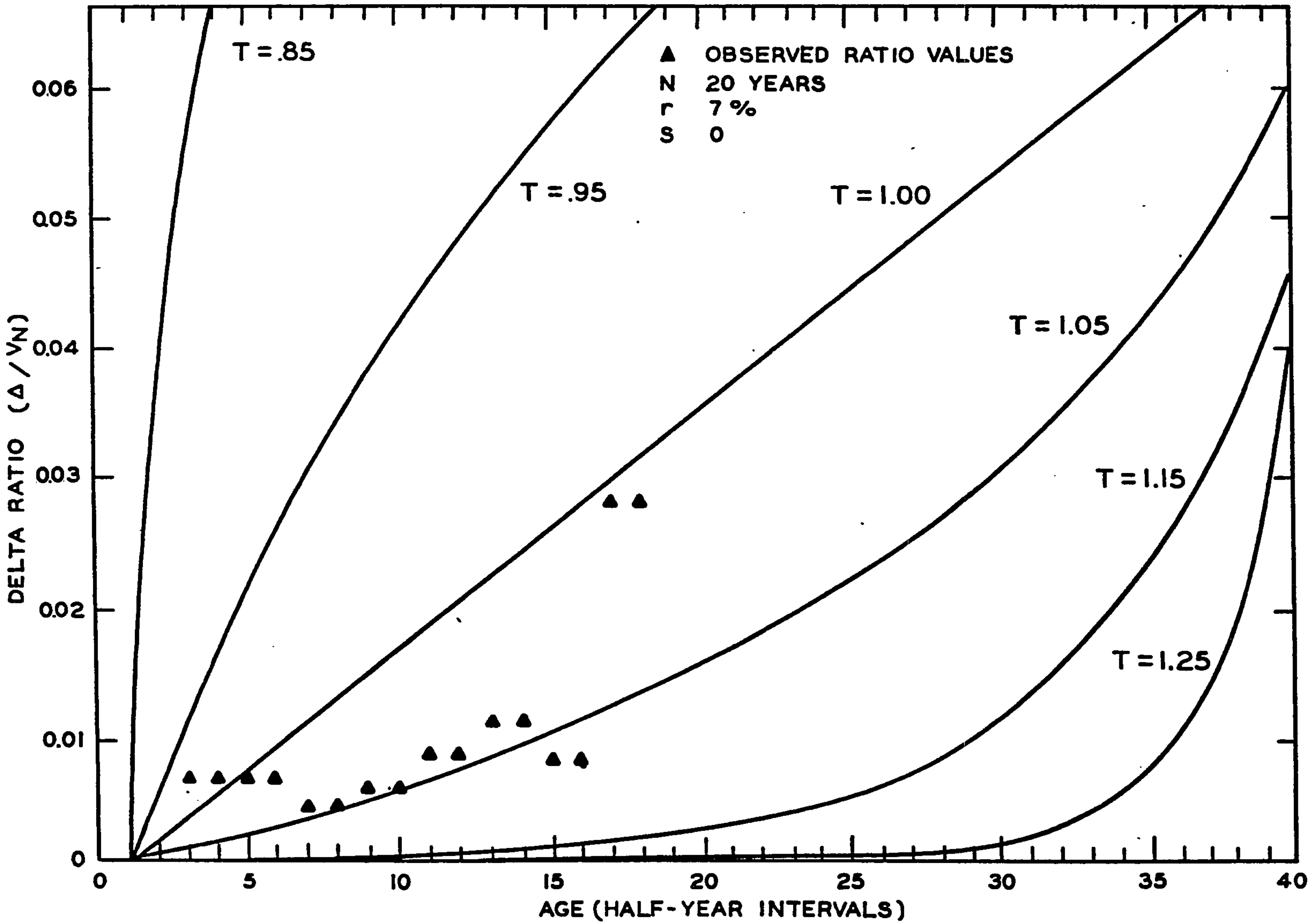

Fig. 26. Standard curves for Property C

Table 20. Computation of Delta Procedure parameters for Property D

Age (yrs)	P ($)	Delta (Δ)	Delta ratio
0.5	3739	0	.000
1.0	3740	1	.000
1.5	3712	- 27	----
2.0	3712	- 27	----
2.5	6681	2942	.064
3.0	6682	2943	.064
3.5	5776	2037	.044
4.0	5777	2038	.044
4.5	6085	2346	.051
5.0	6086	2347	.051
5.5	5826	2087	.045
6.0	5827	2088	.045
6.5	7361	3622	.078
7.0	7361	3622	.078
7.5	7069	3330	.072
8.0	7069	3330	.072
8.5	5594	1855	.040
9.0	5594	1855	.040
9.5	6471	2732	.059
10.0	6471	2732	.059
10.5	3242	- 497	----
11.0	3243	- 496	----

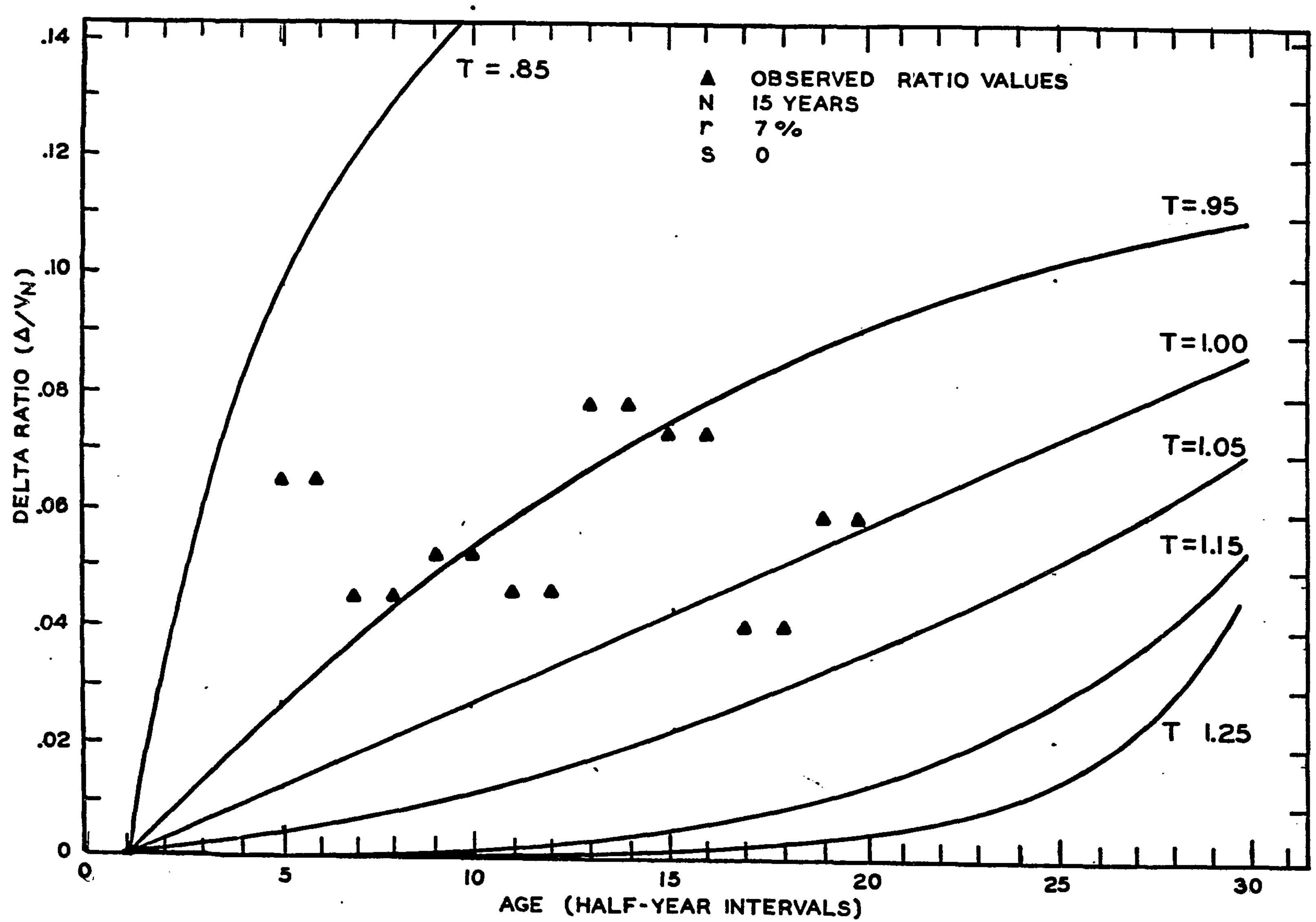

Fig. 27. Standard curves for Property D

.95 was estimated to be the best value. Since this property is known to incur significant downtime that was not reflected in the P term quantities, the final T-factor value selected was judgementally adjusted downward to a range of .90 to .95 for Property D.

Property E

Analysis of data collected, and estimation of the progression rate, for Property E produced the following results.

Quantify P   Average annual repair and maintenance expenditures for Property E were previously given in Table 12 and Figure 15. Based on these data, the P term quantities shown in Table 21 were computed for this property.

Compute Δ Results of the Δ term computation are shown in Table 21.

Determine V_N Since all three units were purchased at the same time for the same cost, the original cost for one equals the average cost for all. Trending this original cost using a company supplied index results in a value new equal to $117,833.

Compute Δ/V_N Computational results for the Δ/V_N term are shown in Table 21.

Determine the Probable Service Life   The probable service life has been previously estimated to be about 10 years.

Estimate the Value of T   The standard curve set produced using the Delta Procedure equations is shown in Figure 28. As was done for the

Table 21. Computation of Delta Procedure parameters for Property E

Age (yrs)	P ($)	Delta (Δ)	Delta ratio
0.5	16,707	0	.000
1.0	16,708	1	.000
1.5	14,899	- 1808	----
2.0	14,900	- 1809	----
2.5	24,441	7734	.066
3.0	24,441	7734	.066
3.5	20,170	3463	.029
4.0	20,170	3463	.029
4.5	23,259	6552	.056
5.0	23,259	6552	.056
5.5	27,835	11,128	.094
6.0	27,835	11,128	.094
6.5	20,917	4210	.036
7.0	20,918	4211	.036
7.5	41,597	24,890	.211
8.0	41,598	24,891	.211
8.5	31,333	14,626	.124
9.0	31,334	14,626	.124

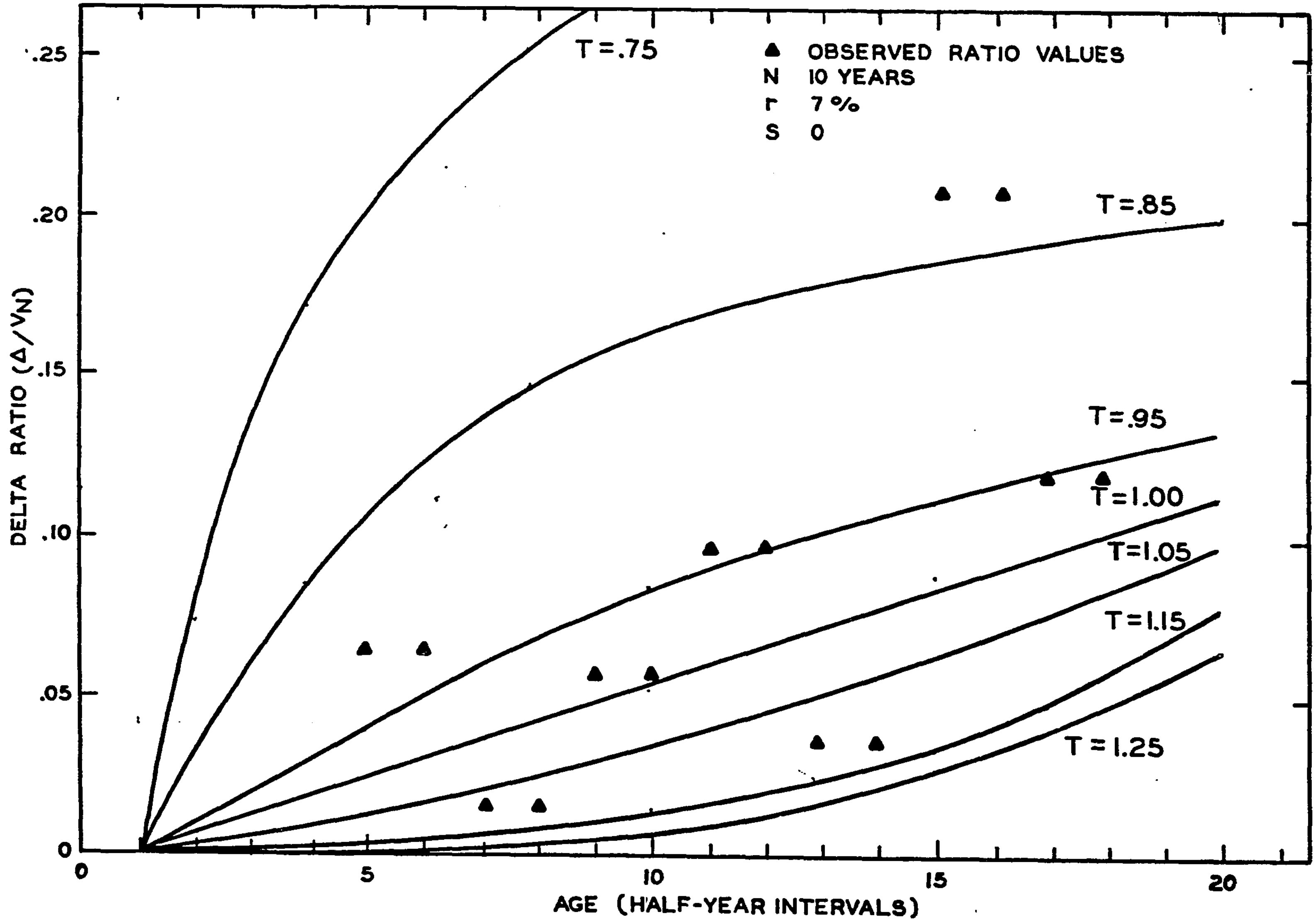

Fig. 28. Standard curves for Property E

other properties, the observed Δ/V_N ratios were plotted on the standard curve set. The T-factor resulting from this procedure was estimated to be in the range of .95 to 1.00 for Property E.

Summary of T-factor Values

The estimated T-factor values or range of values are summarized in Table 22 for the five properties analyzed.

Table 22. Summary of estimated T-factor values for Properties A thru E

Property code	Property description	Number of units	T-factor values
A	die cast machines	16	1.06 - 1.08
B	trim presses	17	1.00 - 1.05
C	printing presses	10	1.05
D	platform trucks	26	.90 - .95
E	forklift trucks	3	.90 - 1.00

Values at Any Age for Iowa Properties

The values at any age x for Properties A through E were computed and tabulated in Table 23. The values were computed using the T-factor values in Table 22 and the equations derived by Elfar.

Table 23. Computed values using the Delta Procedure for Properties A thru E

Age	Property A (die cast machines)				Property B (trim presses)				Property C (printing presses)	
(yrs)	$	%	$	%	$	%	$	%	$	%
0	49963	100	49963	100	18884	100	18884	100	1,126,703	100
1	48863	98	49005	98	17943	95	18413	98	1,066,537	95
2	47650	95	47988	96	17011	90	17920	95	1,004,681	89
3	46400	93	46908	94	16091	85	17404	92	941,037	84
4	45087	90	45764	92	15181	80	16865	89	875,946	78
5	43707	87	44551	89	14286	76	16302	86	809,552	72
6	42261	85	43268	87	13403	71	15716	83	742,095	66
7	40746	82	41912	84	12534	66	15106	80	673,870	60
8	39162	78	40480	81	11680	62	14472	77	605,228	54
9	37509	75	38970	78	10843	57	13816	73	536,586	48
10	35787	72	37381	75	10023	53	13136	70	468,437	42
11	33996	68	35712	71	9222	49	12435	66	401,358	36
12	32140	64	33961	68	8441	45	11713	62	336,023	30
13	30220	60	32130	64	7682	41	10972	58	273,215	24
14	28240	56	30219	60	6945	37	10213	54	213,837	19
15	26206	52	28232	56	6233	33	9439	50	158,932	14
16	24123	48	26172	52	5547	29	8652	46	109,698	10
17	22000	44	24044	48	4889	26	7857	42	67,507	6
18	19847	40	21859	44	4261	23	7056	37	33,927	3
19	17677	35	19625	39	3665	19	6256	33	10,747	1
20	15505	31	17358	35	3103	16	5462	29	0	0
21	13350	27	15075	30	2578	14	4679	25		
22	11233	22	12800	26	2093	11	3918	21		
23	9180	18	10560	21	1650	9	3185	17		
24	7223	14	8390	17	1251	7	2493	13		
25	5398	11	6334	13	901	5	1853	10		
26	3747	8	4443	9	603	3	1279	7		
27	2319	5	2780	6	359	2	787	4		
28	1172	2	1421	3	175	1	396	2		
29	374	1	458	1	54	0	125	1		
30	0	0	0	0	0	0	0	0		

Property D (platform trucks)				Property E (forklift trucks)				Age
$	%	$	%	$	%	$	%	(yrs)
46174	100	46174	100	117833	100	117833	100	0
36358	79	39121	85	93691	80	97906	83	1
28450	61	32860	71	72851	62	79473	67	2
22090	48	27321	59	55060	47	62639	53	3
16987	37	22442	49	40090	34	47514	40	4
12907	28	18166	39	27742	24	34219	29	5
9657	21	14444	31	17843	15	22882	19	6
7085	15	11232	24	10240	9	13639	12	7
5065	11	8489	18	4803	4	6638	6	8
3498	8	6181	13	1421	1	2036	2	9
2302	5	4277	9	0	0	0	0	10
1412	3	2751	6					11
774	2	1579	3					12
348	1	741	2					13
99	0	219	1					14
0	0	0	0					15

Discussion of Select Points Related to the Proposed Delta Procedure

Application of the proposed procedure to the Iowa properties highlighted at least two points needing further discussion. In addition, one of the four original objectives of this study, development of a standardized set of curves, has not yet been attained. Therefore, discussion of these three items will be presented prior to concluding this dissertation.

Standardized Curves

The proposed Delta Procedure depended on using a set of standard curves to estimate a value for the T-factor. These curves were produced by plotting values of Δ/V_N computed using Equation 34, or one of the special cases if applicable, against age. Each set of curves was computed based on specified values of probable service life (N), salvage ratio (S), and discount rate (r) with the T-factor being incremented for each series of calculations. Standard curves for a variety of common situations were produced, and are given in Appendix B. These curves may be used to estimate T-factor values for properties with similar parameter values. If the appropriate curve is not contained in Appendix B, then the computer program listed in Appendix C may be used to compute the proper curve set.

In addition to producing a series of standard curve sets, a test was run to determine what the relative effects would be if the values of N, S, and r were varied. The curves resulting from varying the probable service life were plotted as Figure 29. The horizontal axis was plotted

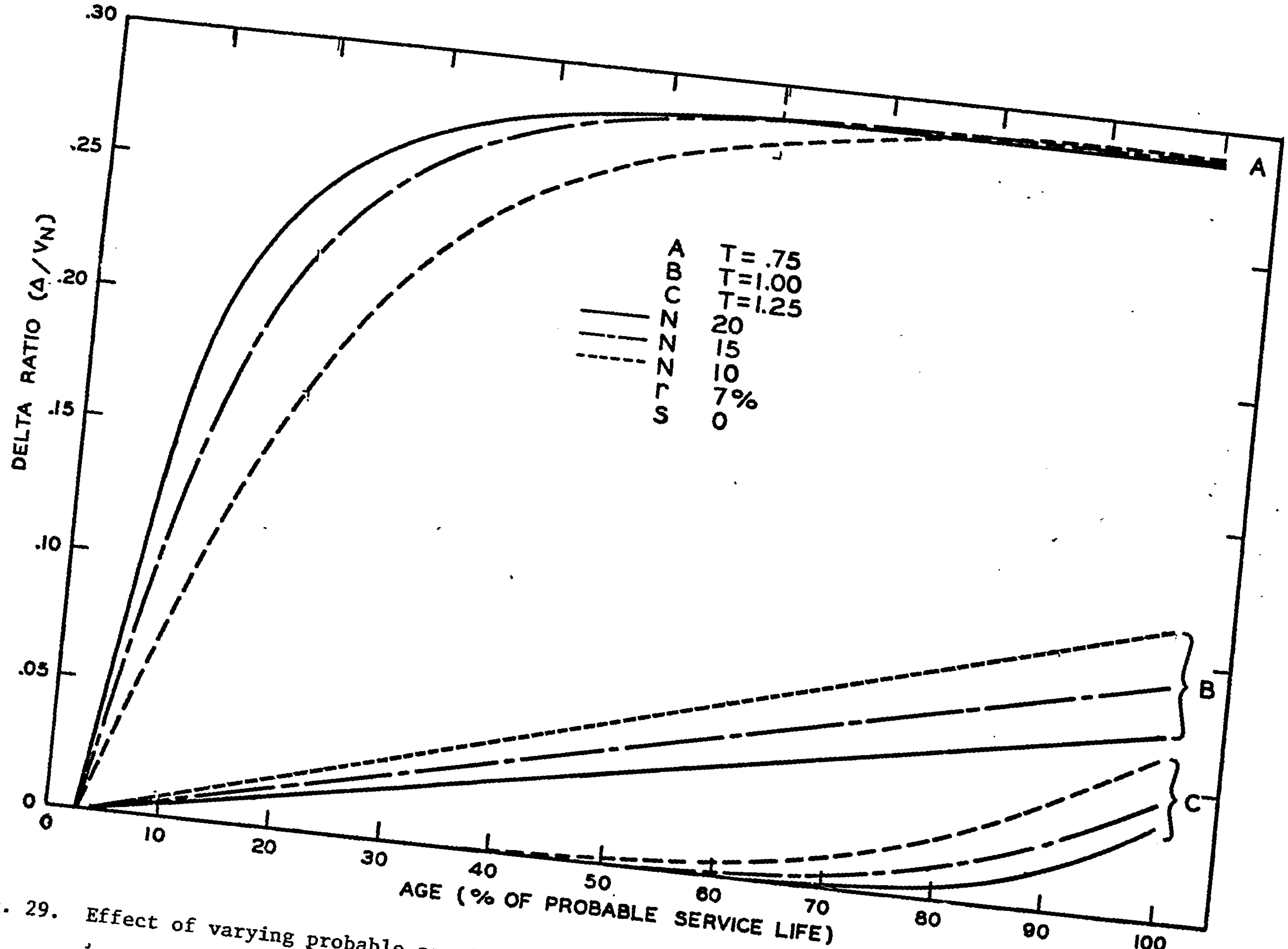

Fig. 29. Effect of varying probable service life on standard curves

in units of % of probable service life, rather than years, so that the results could be presented in consistent terms.

The variation in curve shape that resulted from substituting different salvage ratio values was plotted in Figure 30. The salvage ratio for these curves was incremented from 0 to .10 to .20 in value.

Finally, Figure 31 was produced to demonstrate the effect of varying the discount rate from 0% to 7% to 14% to 21% in magnitude.

Examination of Figures 29 and 30 revealed that varying the values of probable service life and salvage ratio have minimal effect on the standard curve obtained. The slight shift in curve location was minor compared to the rather large variances normally contained in the observed data. The effect of significantly changing the discount rate was shown in Figure 31 to be substantial, however. Therefore, selection of a value for the discount rate should be made with some care. Use of an inflation-free rate of return, as was recommended previously, should simplify the selection somewhat.

Annual vs. Semi-annual Interval Length

Elfar's original derivation was based on an interval length of one-half year. The semi-annual interval length was selected so that property values at half-year ages could be computed and so that the usually nonuniform cash flows could be represented more accurately (14, p. 45). The half-year convention was retained in this study for sake of uniformity. In many cases, however, the available data are summarized on an annual basis. Therefore, the effect of defining interval length in whole-year units rather than semi-annual units was explored.

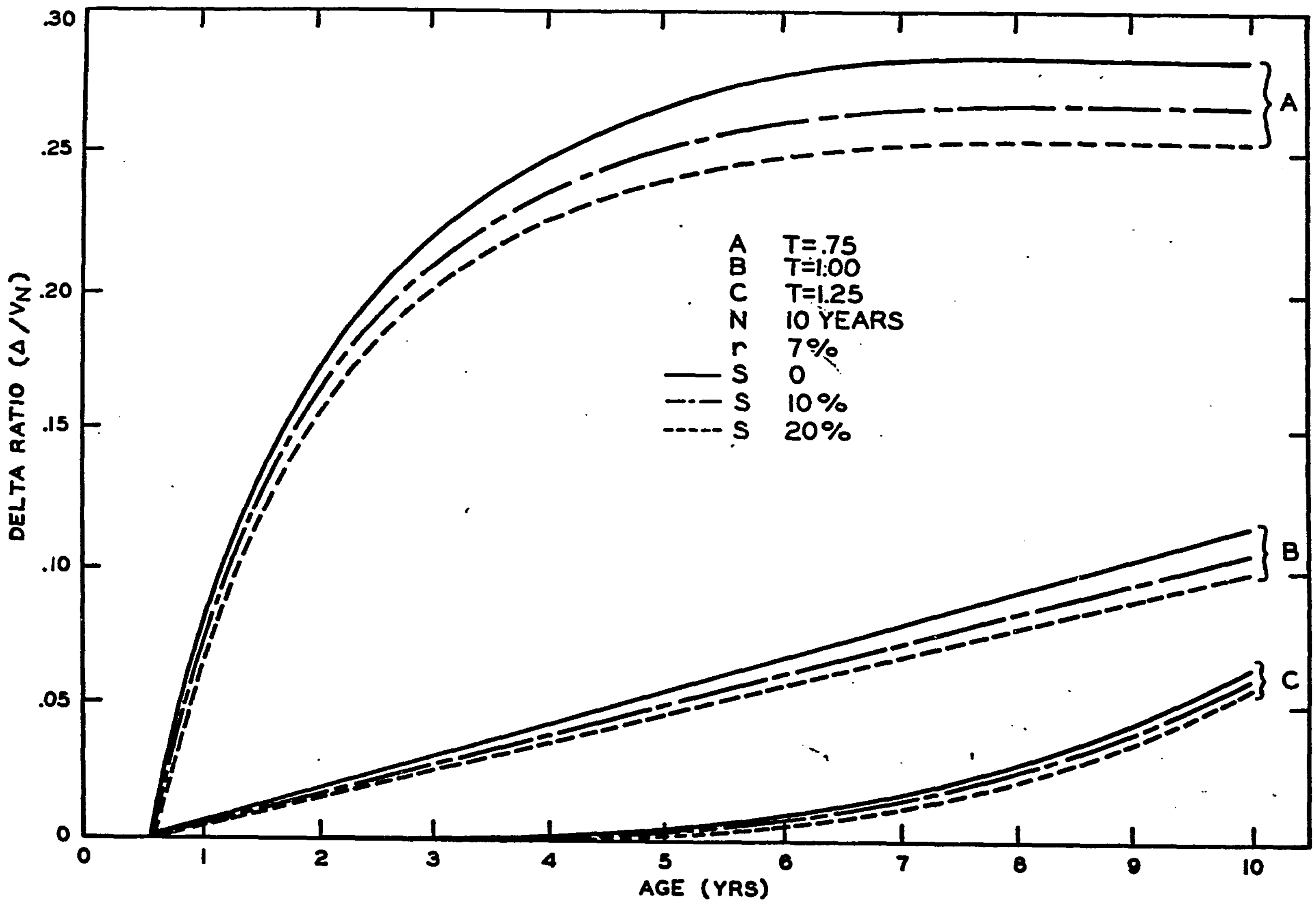

Fig. 30. Effect of varying salvage ratio on standard curves

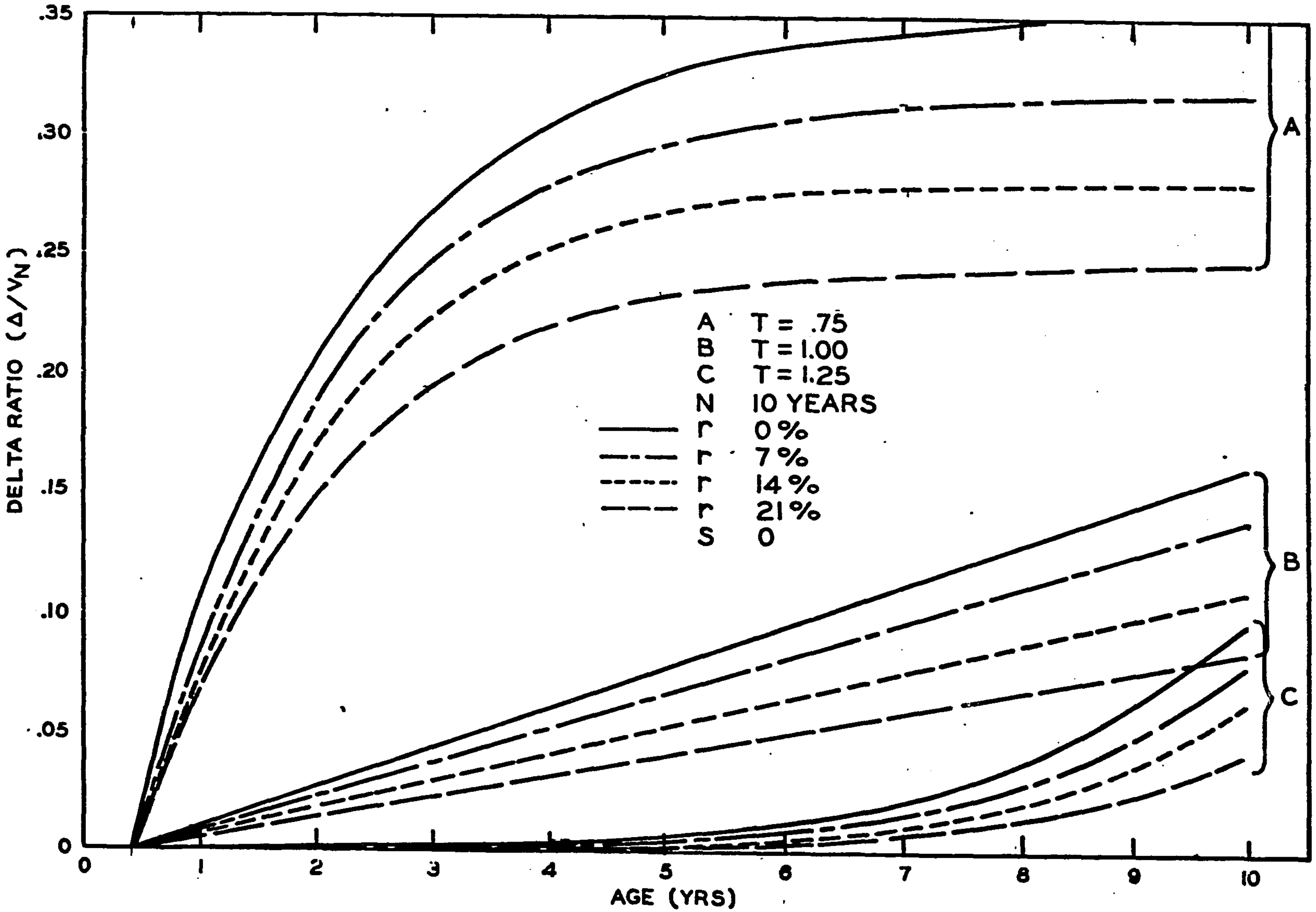

Fig. 31. Effect of varying the rate of return on standard curves

A review of the derivations leading to Equations 8, 9, and 34 revealed that redefinition of the interval length, as long as the other parameters were also redefined in consistent units, did not change the resulting expressions. Likewise, none of the resulting special case equations were dependent on interval length chosen. Actual substitution of values into Equation 34, however, did produce a different set of standard curves. The extent of the difference is illustrated in Figure 32. Parameter values used to compute these curves were N = 20 years, r = 7%, and S = 10%. As can be seen, the curve set based on an annual interval length had generally higher values of the Δ/V_N ratio for a given age and T-factor value than does the set based on semi-annual interval lengths.

Since the parameter values used for Figure 32 corresponded to those identified for the D8 dozer property previously analyzed, a comparison was made between T-factor values estimated for D8 dozers using both curve sets. The observed Δ/V_N ratios for both the annual and semi-annual interval lengths were computed, then plotted on Figure 32. Based on the semi-annual interval length, the best T-factor value had previously been estimated to fall in a range of .96 to .98. Based in an annual interval length, the best T-factor value was estimated to be in a range of .93 to .97. Due to the inexact nature of much of the input data, and the approximate nature of the procedure, the difference in estimated T-factor values was considered to be insignificant. Therefore, though the semi-annual interval length may have resulted in some increased convenience by having property values at half-year intervals, any resultant increase in

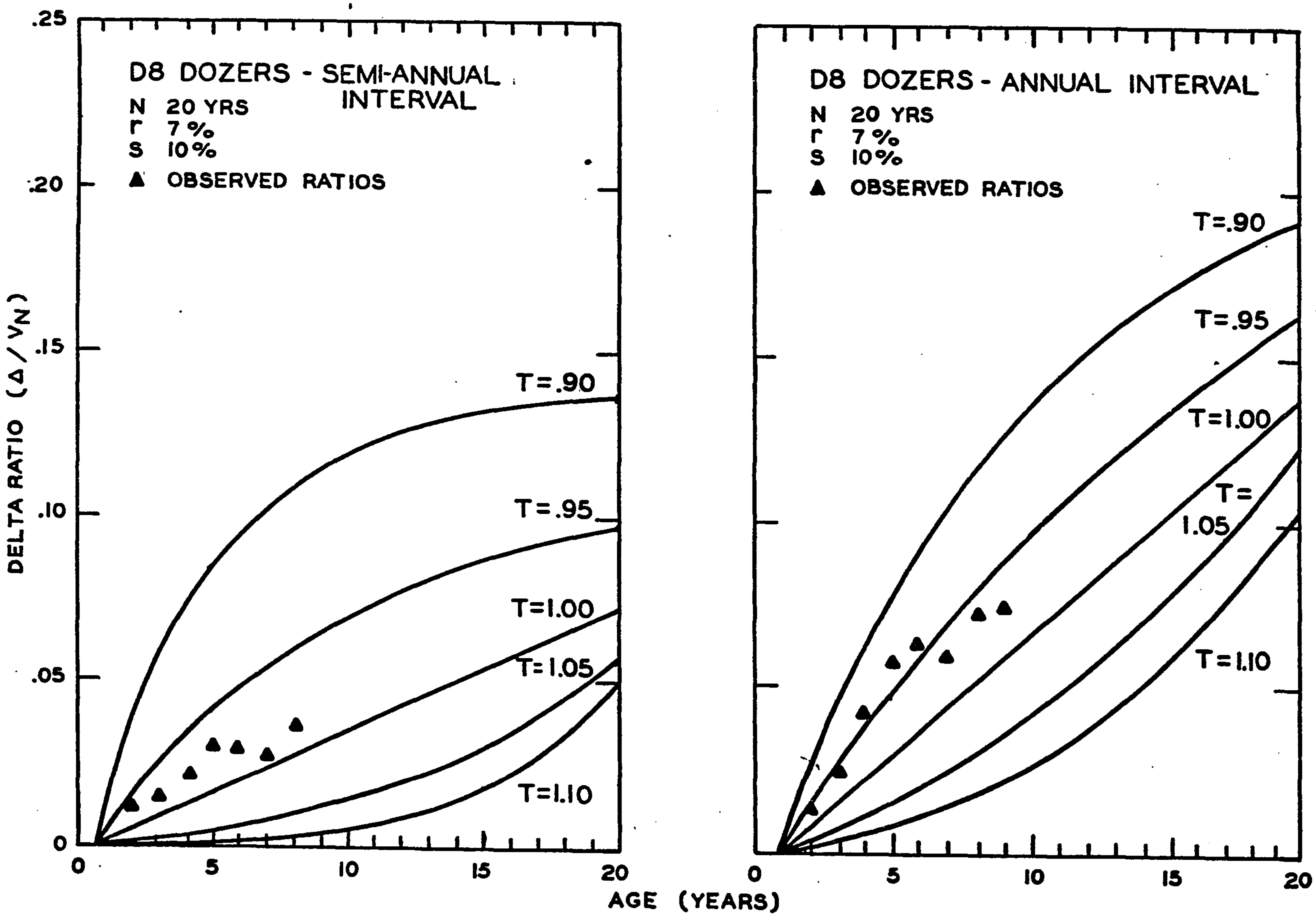

Fig. 32. Comparison of semi-annual and annual interval lengths

accuracy appeared to be minimal.

The Need for Judgement

The final point needing further discussion was the role of experienced judgement in the proposed procedure. Though the derivations presented were quite exact, and the resulting equations were precise in form, the data input and results obtained were not necessarily complete or completely reliable. As in all valuation situations, judgement must be exercised in the collection, interpretation, and utilization of the input data and in the interpretation of of the results obtained. The procedures proposed by this dissertation are intended to augment, not replace, the role of judgement in current valuation practice.

CONCLUSIONS

The following conclusions may be drawn from this study:

1. The Elfar model appears to be valid. Verification of this model was shown by its ability to predict value curves that closely fit observed market evidence data.

2. Progression rate values may be estimated using the Ratio Procedure if appropriate data are available to measure the operation returns for the new vs. the subject property.

3. Progression rate values may be estimated using the Delta Procedure if appropriate data are available to measure the increasing amounts of annual depreciation.

4. The fair rate of return selected has a significant effect on the standard curves computed using the Delta Procedure, and therefore the T-factor estimated for a property account. The determination of probable service life and salvage ratio, on the other hand, appears to have much less effect on the outcome.

5. Interval length used in the application of the Delta Procedure may be either annual or semi-annual. As long as a consistent basis is used throughout the analysis, no significant difference in results is apparent.

6. The exercise of judgement in the interpretation of results obtained using the Ratio Procedure or Delta Procedure is still required.

Judgement is particularly needed where insufficient or inappropriate data exists, or where the loss in operation returns cannot be accurately or completely quantified.

7. Further study is needed in areas associated with this dissertation. Specifically, it is suggested that further work be pursued in the following topics:

a. Extensive data collection and analysis should be undertaken in an effort to identify progression rate values typical of various types of equipment or characteristic of various classes of industries.

b. The causes, effects, and magnitudes of start-up or shakedown costs for an equipment unit should be explored. A method of properly incorporating these costs into the Ratio or Delta Procedures should be devised to improve the results.

c. The application of the Ratio or Delta Procedures to composite properties should be studied. Composite properties commonly occur when standardized recordkeeping systems are required by regulatory bodies or when an entire industrial plant composed of many discrete and dissimilar units is being valued. Modifications to the proposed procedures to accurately handle these properties are needed.

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APPENDIX A

Table A1. Income statement for Alpha-1 refinery: 100,000 barrel/day, 5 year old plant vs. modern replacement[a]

	5 year old plant	difference	modern replacement
revenue		2.9	
costs			
raw materials		0	
operating costs[b]		0.3	
depreciation		(2.0)	
taxable income	12.0	1.2	13.2
taxes (federal)	5.8		6.4
net income	6.2		6.9
depreciation	5.8		7.8
cash income after tax	12.0		14.7[c]

[a] All figures in units of $MM/YR

[b] Excludes own consumption fuel

[c] Cash income after taxes needed to generate 7 percent real rate of return on 156 $MM investment with 20 year economic life

Table A2. Income statement for Alpha-2 refinery: 100,000 barrel/day, 10 year old plant vs. modern replacement[a]

	10 year old plant	difference	modern replacement
revenue		5.1	
costs			
raw materials		0	
operating costs[b]		2.2	
depreciation		(4.0)	
taxable income	9.9	3.3	13.2
taxes (federal)	4.8		6.4
net income	5.1		6.9
depreciation	3.8		7.8
cash income after tax	8.9		14.7[c]

[a] All figures in units of $MM/YR

[b] Excludes own consumption fuel

[c] Cash income after taxes needed to generate 7 percent real rate of return on 156 $MM investment with 20 year economic life

Table A3. Income statement for Alpha-3 refinery: 100,000 barrel/day, 16 year old plant vs. modern replacement[a]

	16 year old plant	difference	modern replacement
revenue		7.5	
costs			
raw materials		0	
operating costs[b]		3.4	
depreciation		(6.0)	
taxable income	8.4	4.9	13.2
taxes (federal)	4.0		6.4
net income	4.4		6.9
depreciation	1.8		7.8
cash income after tax	6.2		14.7[c]

[a] All figures in units of $MM/YR

[b] Excludes own consumption fuel

[c] Cash income after taxes needed to generate 7 percent real rate of return on 156 $MM investment with 20 year economic life

Table A4. Income statement for Beta refinery: 100,000 barrel/day, 10 year old plant vs. modern replacement[a]

	10 year old plant	difference	modern replacement
revenue		12.9	
costs			
raw materials		0	
operating costs[b]		6.9	
depreciation		(4.0)	
taxable income	5.2	15.8	21.0
taxes (federal)	2.5		10.1
net income	2.7		10.9
depreciation	8.3		12.3
cash income after tax	11.0		23.2[c]

[a] All figures in units of $MM/YR

[b] Excludes own consumption fuel

[c] Cash income after taxes needed to generate 7 percent real rate of return on 245 $MM investment with 20 year economic life

Table A5. Income statement for Gamma refinery: 300,000 barrel/day, 17.3 year old plant vs. modern replacement[a]

	17.3 year old plant	difference	modern replacement
revenue		39.0	
costs			
raw materials		0	
operating costs[b]		21.9	
depreciation		(25.5)	
taxable income	9.4	35.4	44.8
taxes (federal)	4.5		21.5
net income	4.9		23.3
depreciation	12.9		38.4
cash income after tax	17.8		61.7[c]

[a] All figures in units of $MM/YR

[b] Excludes own consumption fuel

[c] Cash income after taxes needed to generate 7 percent real rate of return on 768 $MM investment with 20 year economic life

APPENDIX B

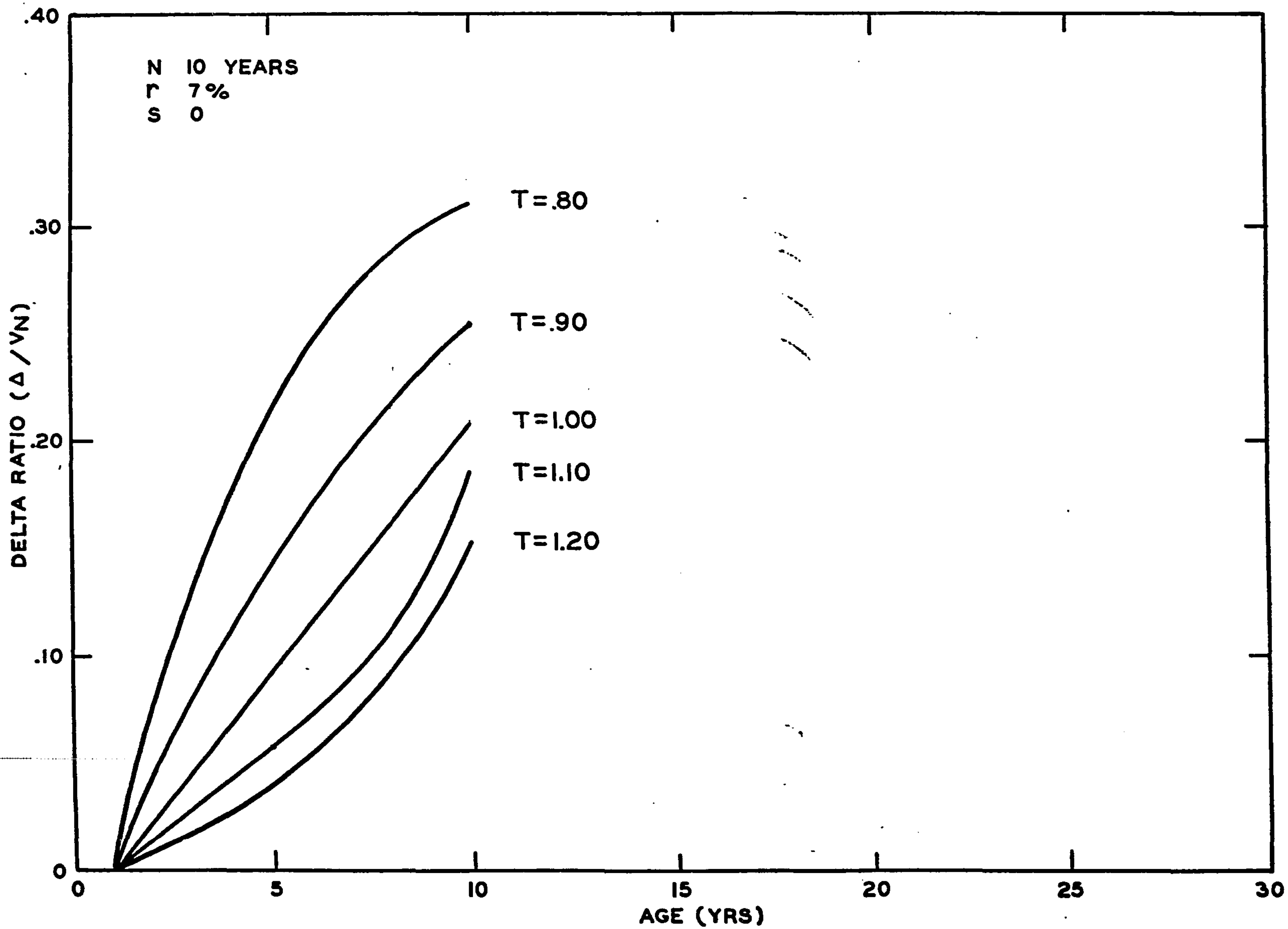

Fig. B1 - Standard curves: N = 10 years, r = 7%, S = 0

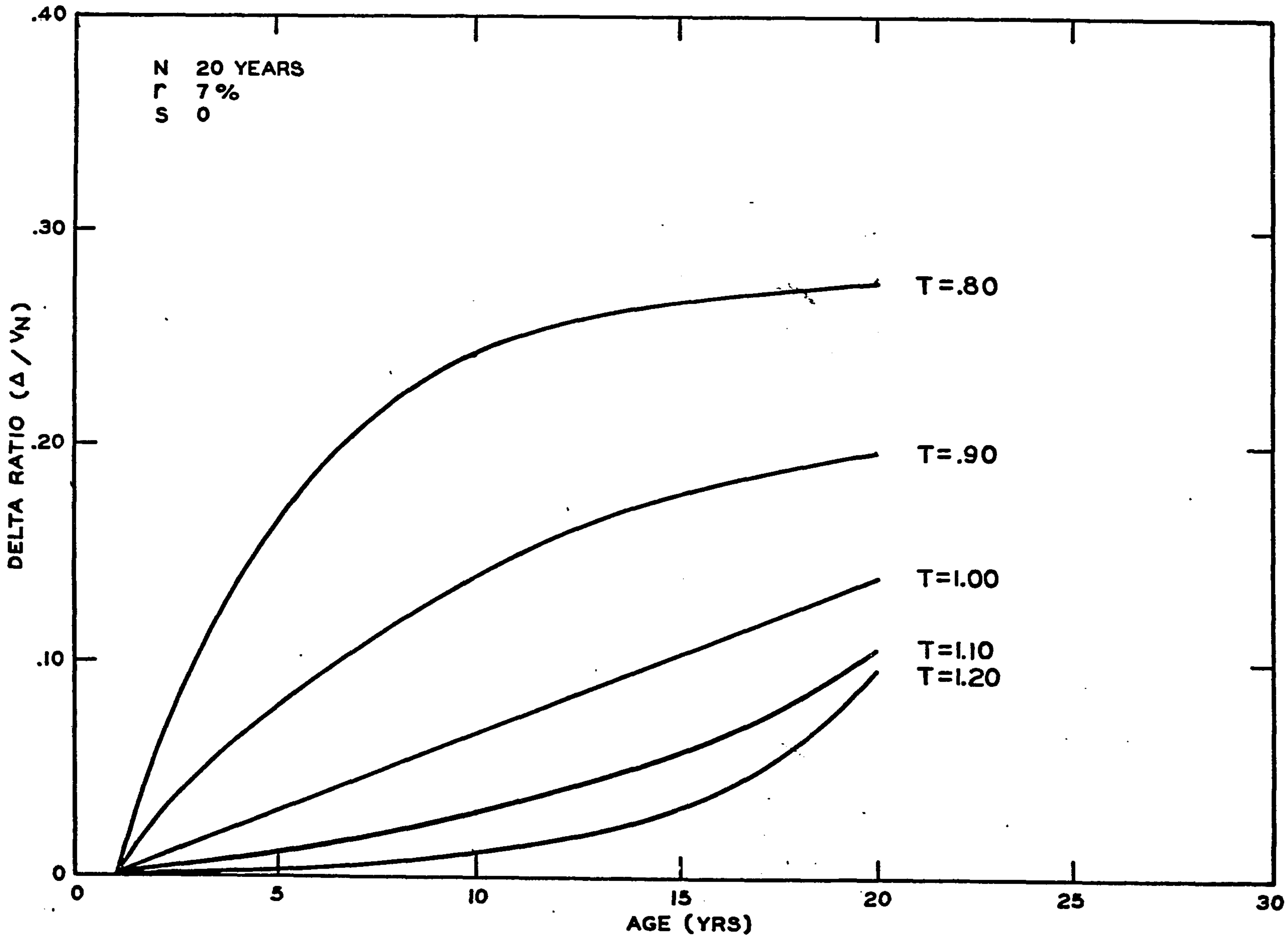

Fig. B2 - Standard curves: N = 20 years, r = 7%, S = 0

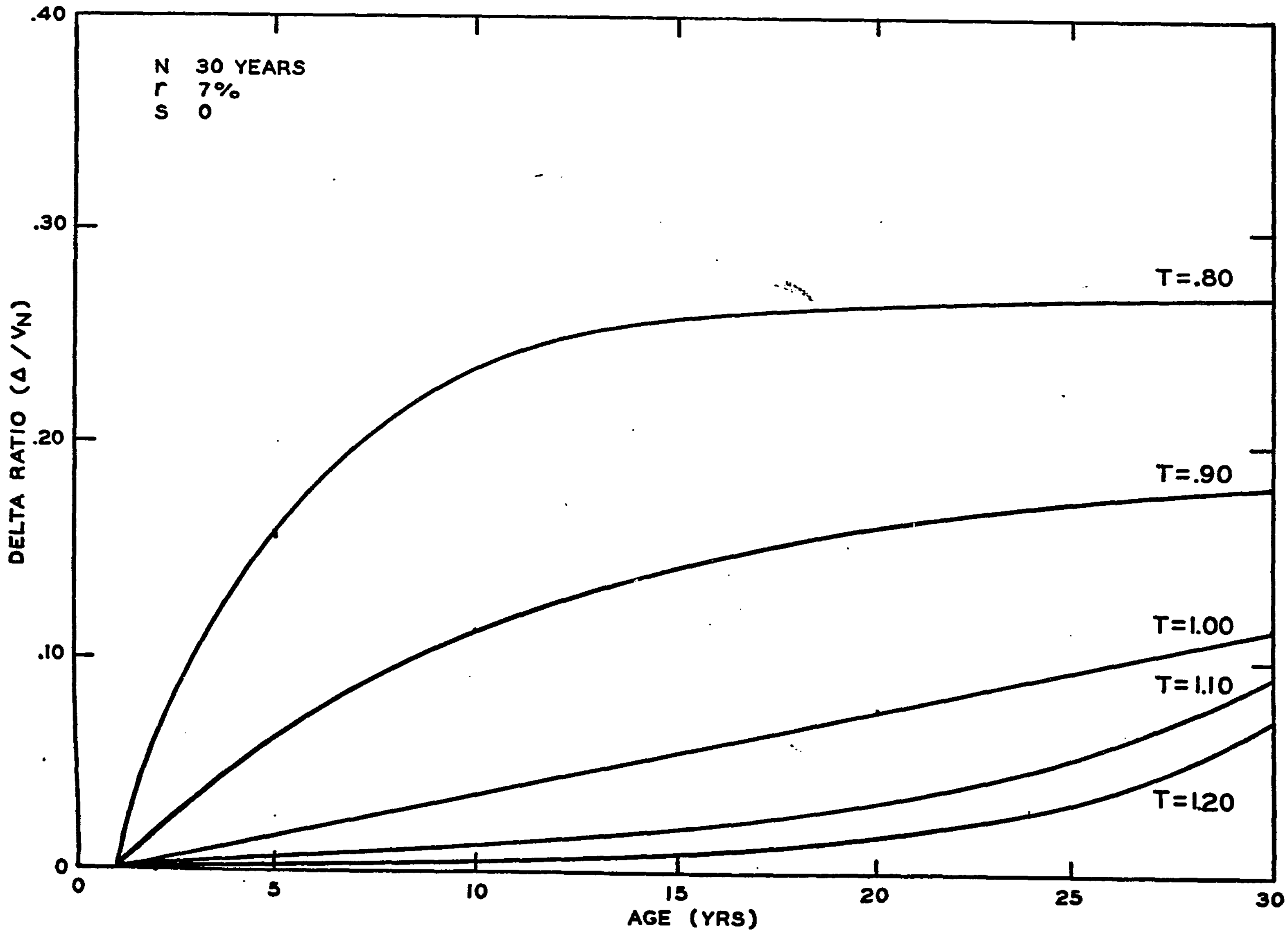

Fig. B3 - Standard curves: N = 30 years, r = 7%, S = 0

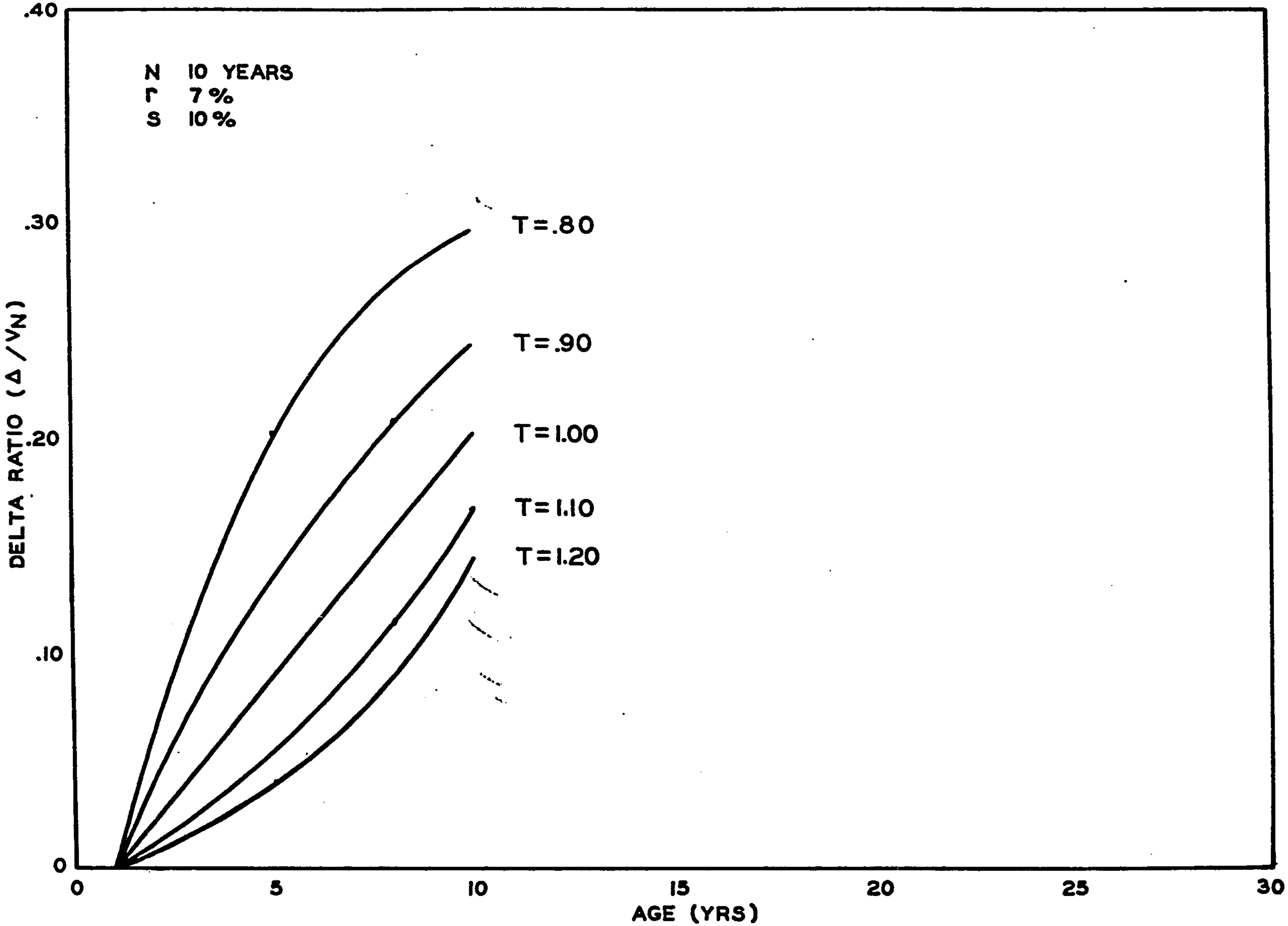

Fig. B4 - Standard curves: N = 10 years, r = 7%, S = 10%

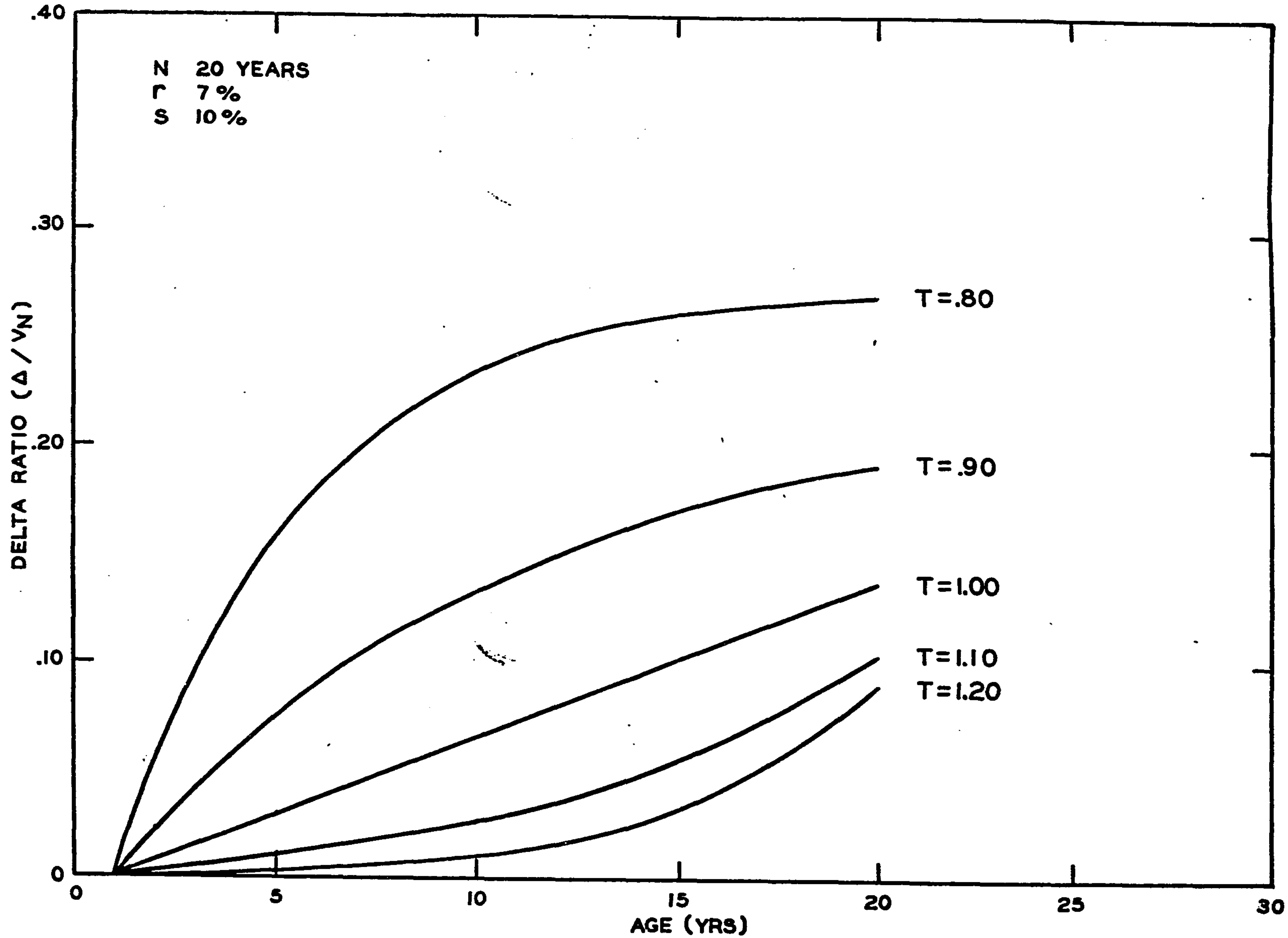

Fig. B5 - Standard curves: N = 20 years, r = 7%, S = 10%

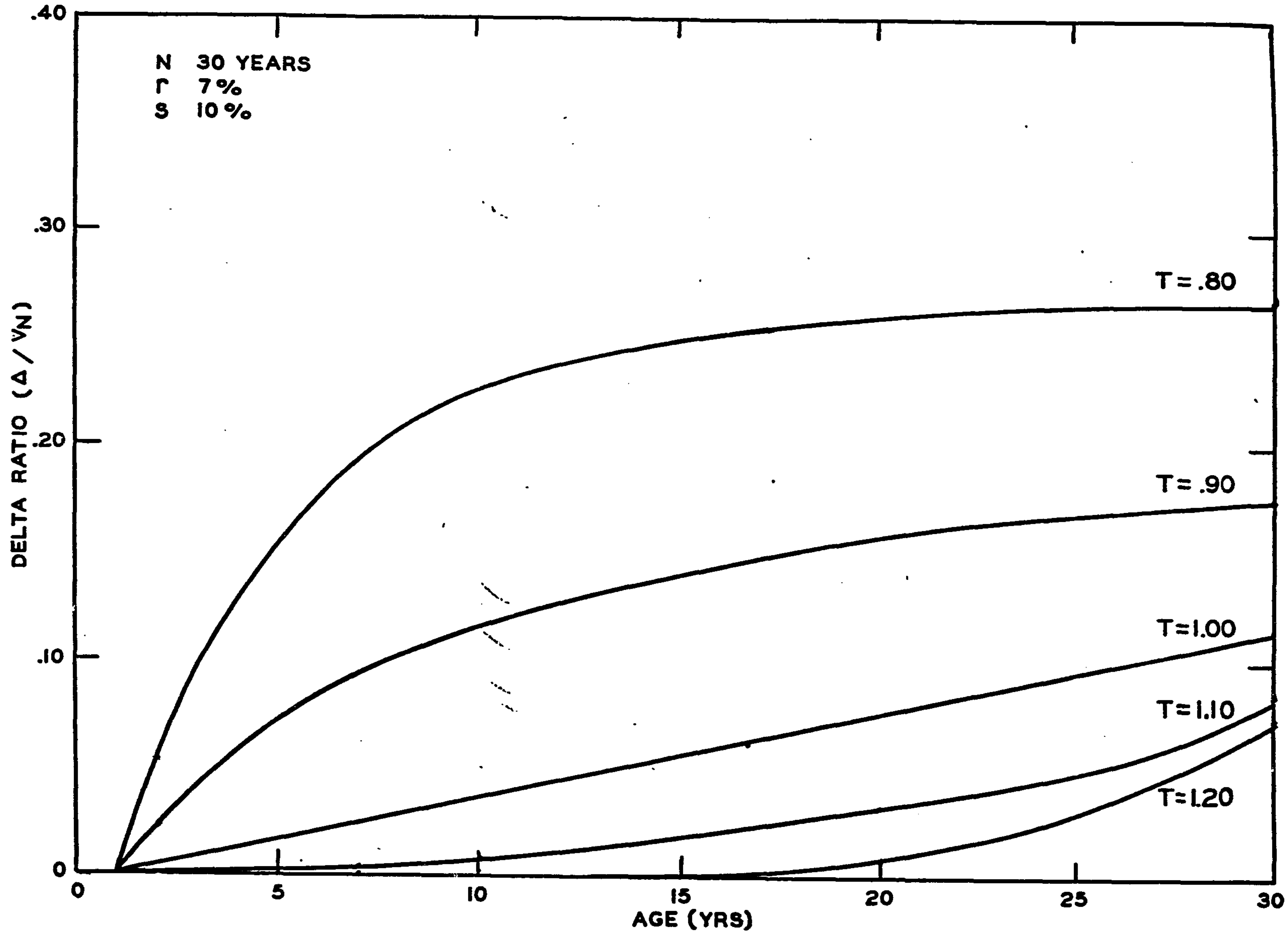

Fig. B6 - Standard curves: N = 30 years, r = 7%, S = 10%

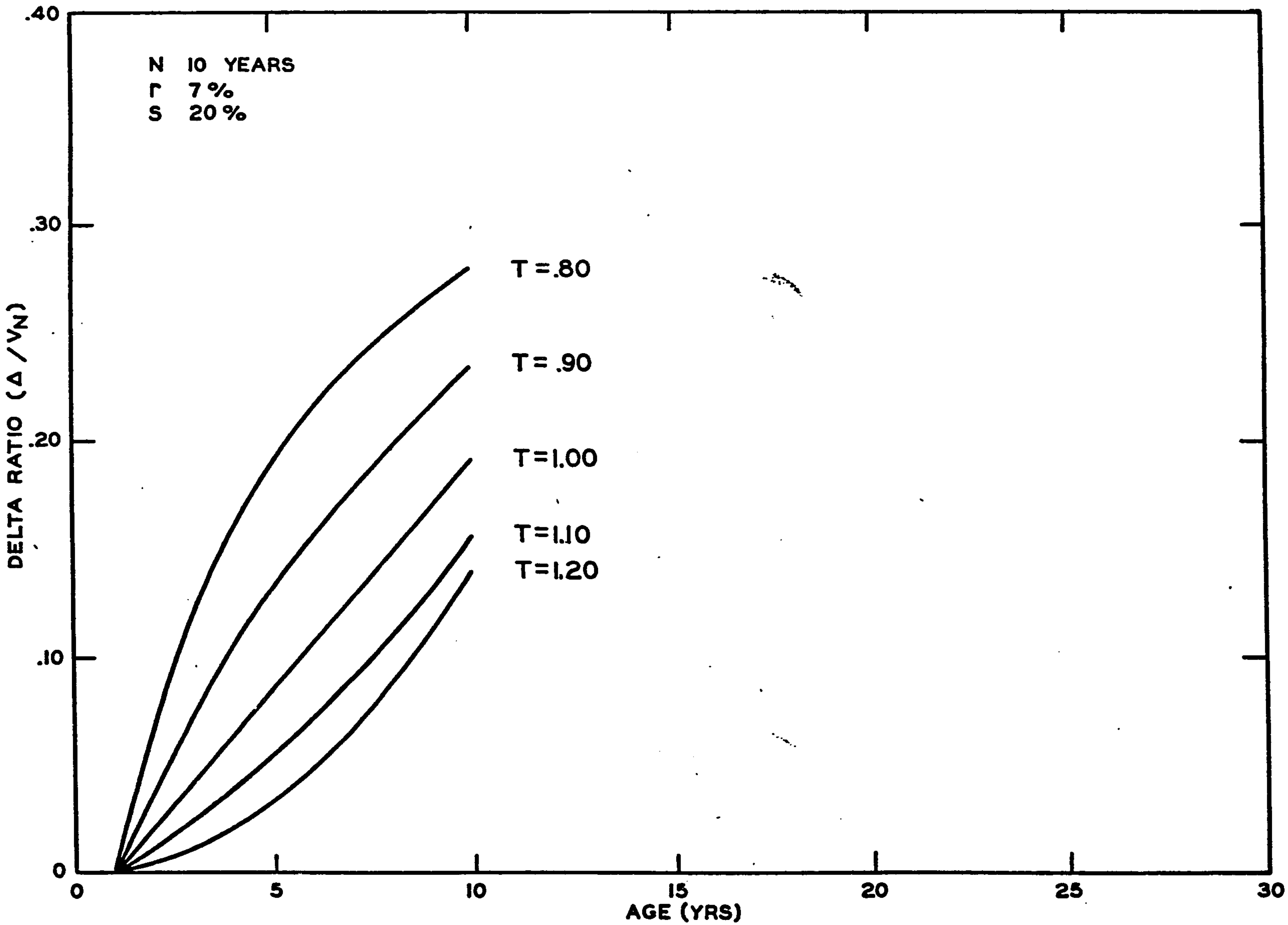

Fig. B7 - Standard curves: N = 10 years, r = 7%, S = 20%

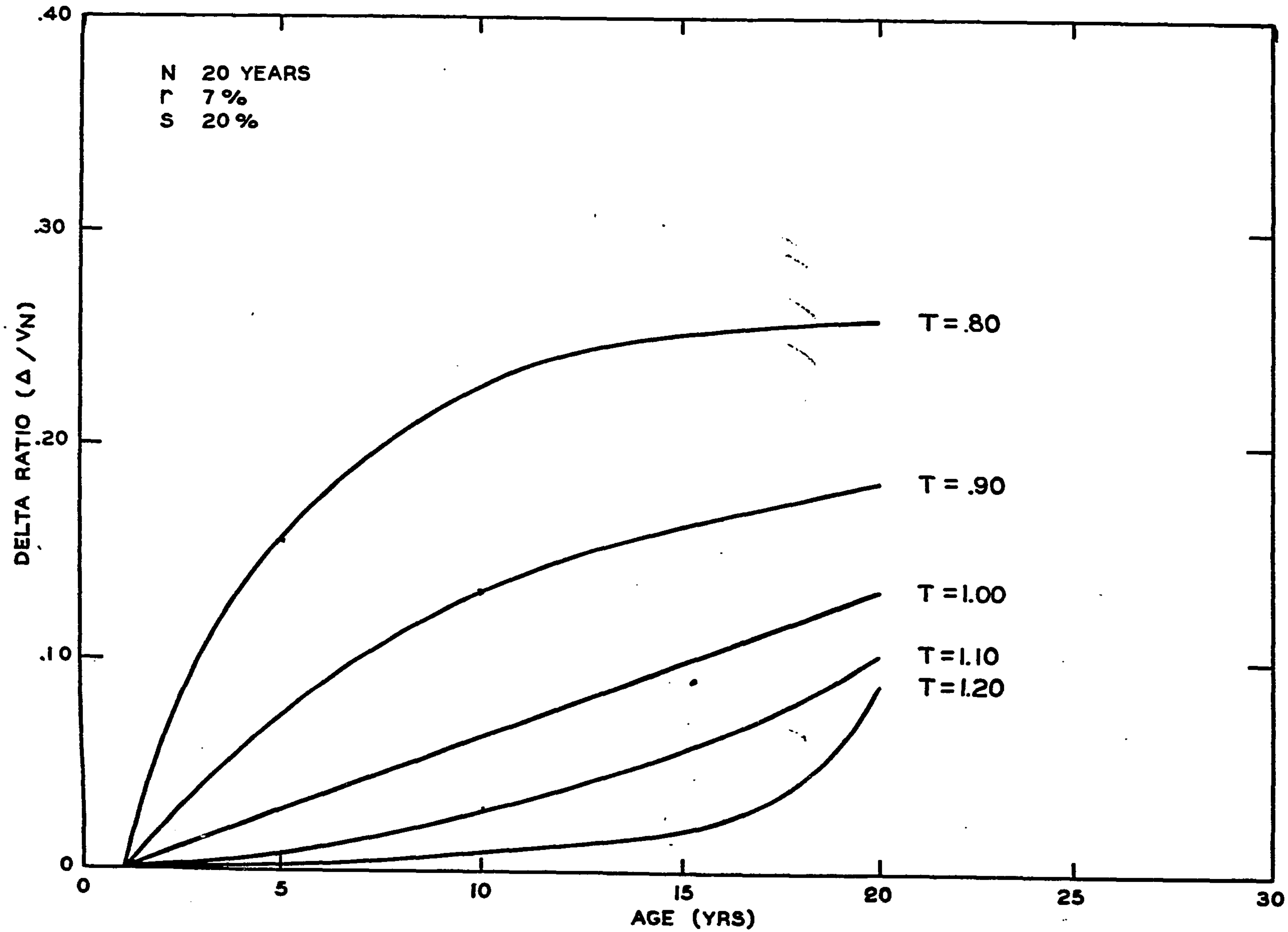

Fig. B8 - Standard curves: N = 20 years, r = 7%, S = 20%

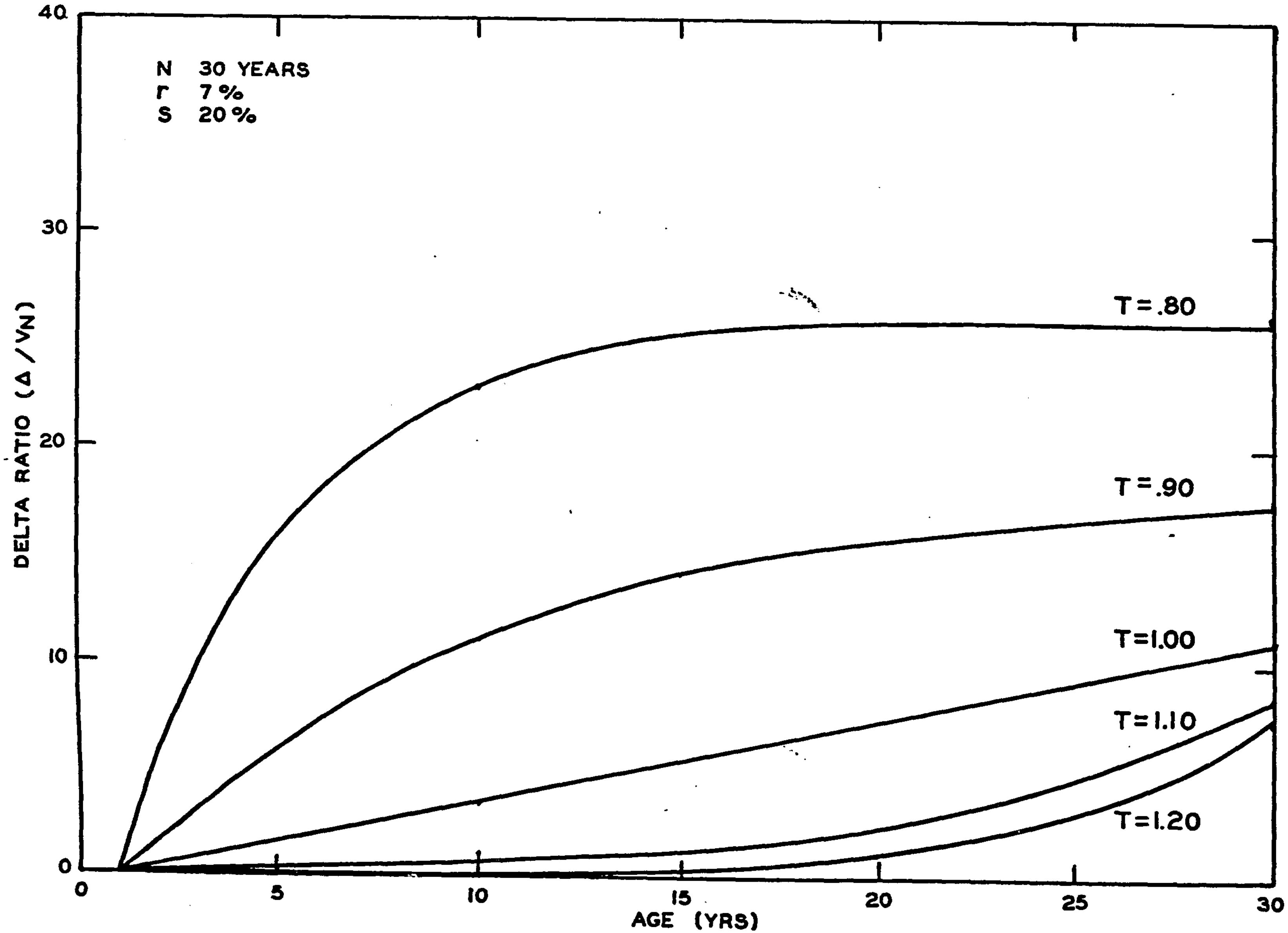

Fig. B9 - Standard curves: N = 30 years, r = 7%, S = 20%

APPENDIX C

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C     APPENDIX C
C
C
C     THIS PROGRAM COMPUTES VALUES FOR THE STANDARD CURVES USED IN THE
C     APPLICATION OF THE DELTA PROCEDURE.  ANY NUMBER OF CURVE SETS MAY
C     COMPUTED IN A SINGLE RUN.  ONE DATA CARD FOR EACH SET IS REQUIRED.
      PROGRAM VALUEX(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION RATIO(15,80)
      INTEGER PL
C     DATA IS READ IN THE FOLLOWING ORDER AND FORMAT:  PERIOD LENGTH
C     (PERLEN) IS IN UNITS OF YEARS , EXPRESSED IN A F5.1 FORMAT, AND
C     LOCATED WITH THE DECIMAL POINT IN COLUMN 4.  RATE OF RETURN (R) IS
C     IN UNITS OF % PER YEAR, EXPRESSED IN A F4.1 FORMAT, AND LOCATED
C     WITH THE DECIMAL POINT IN COLUMN 9.  SALVAGE RATIO (S) IS A
C     DECIMAL, EXPRESSED IN A F4.2 FORMAT, AND LOCATED WITH THE DECIMAL
C     POINT IN COLUMN 13.  PROBABLE SERVICE LIFE (PL) IS IN UNITS OF
C     YEARS, EXPRESSED IN AN I2 FORMAT, AND LOCATED WITH THE LAST
C     DIGIT IN COLUMN 20.  THE PROGRAM CEASES COMPUTATION WHEN NO MORE
C     DATA CARDS ARE FOUND.
    5 READ(5,10)PERLEN,R,S,PL
      IF(EOF(5).NE.0) GO TO 200
   10 FORMAT(F5.1,1X,F4.1,1X,F4.2,3X,I2)
      R=R/100.0
      IF(PERLEN.EQ.1.0) GO TO 15
      R=((1.0+R)**.5)-1.0
      PL=2.0*PL
   15 Q=R+1.0
      N=PL
C     THE PROGRAM COMPUTES 11 CURVES IN A SET.  IF MORE OR LESS THAN
C     THIS NUMBER ARE DESIRED, CHANGE THE NUMBER OF ITERATIONS IN THE
C     FOLLOWING DO LOOP.
      DO 65 I=1,11
C     THE PROGRAM COMPUTES CURVE VALUES FOR T = .75 TO T = 1.25 IN .05
C     INCREMENTS.  IF A DIFFERENT RANGE OR INCREMENT IS DESIRED, CHANGE
C     THE APPROPRIATE VALUES IN THE NEXT STATEMENT.
      T=.70 + (I*.05)
      IF(T.EQ.1.0) GO TO 45
      IF(T.GT.50.0) GO TO 35
      IF(R.EQ.0.0) GO TO 25
C     COMPUTES VALUES FOR GENERAL CASE
      DO 20 K=1,PL
   20 RATIO(I,K)=((Q**N-S)*(T**(K-1)-1.0)*(T-Q)*R)/((T**N)*((T*Q**N)-T-
     9(Q**(N+1))+1.0)+R*(Q**N))
      GO TO 65
C     COMPUTES VALUES FOR R=0% AND 0<T<INFINITY
   25 DO 30 K=1,PL
   30 RATIO(I,K)=((1.0-S)*(T-1.0)*(T**(K-1)-1.0))/((T**N)*(N*T-N-1.)+1.)
      GO TO 65
C     COMPUTES VALUES FOR T=INFINITY
   35 DO 40 K=1,PL
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```
   40 RATIO(I,K)=0
      GO TO 65
   45 IF(R.EQ.0.0) GO TO 55
C     COMPUTES VALUES FOR T=1 AND R>0
      DO 50 K=1,PL
   50 RATIO(I,K)=((R**2.0)*(Q**N-S)*(K-1))/((Q**N)*(N*R-1.0)+1.0)
      GO TO 65
C     COMPUTES VALUES FOR T=1 AND R=0
   55 DO 60 K=1,PL
   60 RATIO(I,K)=(2.0*(1.0-S)*(K-1))/(N*(N+1))
   65 CONTINUE
C     WRITE RESULTS FOR EACH CURVE SET
      WRITE(6,70)PL,S,R,PERLEN
   70 FORMAT("1",10X,"PROBABLE LIFE = ",I2," PERIODS",//,10X,
     9"SALVAGE RATIO = ",F4.2,//,10X,"DISCOUNT RATE = ",F8.5,//,10X,
     9"PERIOD LENGTH = ",F5.1,///,"AGE T= .75 T= .80 T= .85 T= .90 T= .9
     95 T=1.00 T=1.05 T=1.10 T=1.15 T=1.20 T=1.25",//)
      DO 80 K=1,PL
      WRITE(6,75)K,(RATIO(I,K),I=1,11)
   75 FORMAT(1X,I2,11F7.3)
   80 CONTINUE
      GO TO 5
  200 STOP
      END
```